Questions on 5.5 HW?

6) $f(x) = \cos^{-1}(x)$, $a = 0$

$f(0) = \cos^{-1}(0)$
$= \frac{\pi}{2}$
$(0, \pi/2)$

a) $f'(x) = -\frac{1}{\sqrt{1-(x)^2}}$

$f'(0) = \frac{-1}{\sqrt{1}} = -1$

$L(x) = \pi/2 - 1(x - 0)$

$L(x) = \pi/2 - x$

b) $L(a+0.1) = L(0.1) = \pi/2 - 0.1 \approx 1.4708$

$f(a+0.1) = f(0.1) = \cos^{-1}(0.1) \approx 1.4706$

Nov 3-2:38 PM

5.6 Related Rates

Related Rates Problem

~What is it?

Any equation involving 2 or more variables that are diff. functions of time t can be used to find an equation that relates their corresponding rates.

Steps/Strategy to solving one

1. Understand the Problem In particular, identify the variable whose rate of change you seek and the variable (or variables) whose rate of change you know.

2. Develop a Mathematical Model of the Problem Draw pictures (many of these problems involve geometric figures) and label the parts that are important to the problem. Be sure to distinguish constant quantities from variables that change over time. Only constant quantities can be assigned numerical values at the start.

3. Write an equation relating the variable whose rate of change you seek with the variable(s) whose rate of change you know. The formula is often geometric, but it could come from a scientific application.

4. Differentiate both sides of the equation implicitly with respect to time. Be sure to follow all the differentiation rules. The Chain Rule will be especially critical, as you will be differentiating with respect to the parameter t.

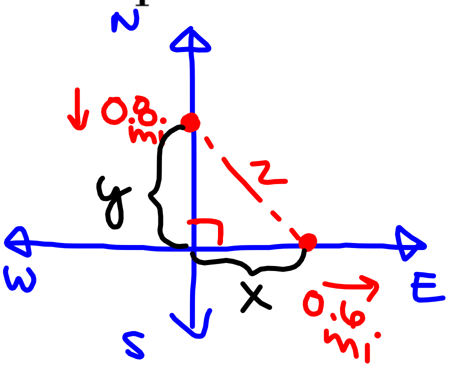
5. Substitute values for any quantities that depend on time. Notice that it is only safe to do this after the differentiation step. Substituting too soon "freezes the picture" and makes changeable variables behave like constants, with zero derivatives.

6. Interpret the Solution Translate your mathematical result into the problem setting (with appropriate units) and decide whether the result makes sense.

Nov 14-10:50 AM

Example

A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.8 mi north of the intersection and the car is 0.6 mi to the east, the police determine with radar that the distance between them and the car is increasing at 15 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?



$0.8^2+0.6^2=z^2$

$1=z^2$

$\boxed{1=z}$

let x = distance of the speeding car from +

y = dist. police car is from +

z = dist. between 2 cars

$\frac{dy}{dt}=-60$ mph, $\frac{dz}{dt}=15$ mph, FIND $\frac{dx}{dt}$

$y=0.8$ mi, $z=1$ mi, $x=0.6$ mi

Use Pythagorean Thm:

$\frac{d}{dx}\left(x^2+y^2=z^2\right)$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$

Plug in: $2(0.6)\frac{dx}{dt}+2(0.8)(-60)=2(1)(15)$

$1.2\frac{dx}{dt}-96=30$

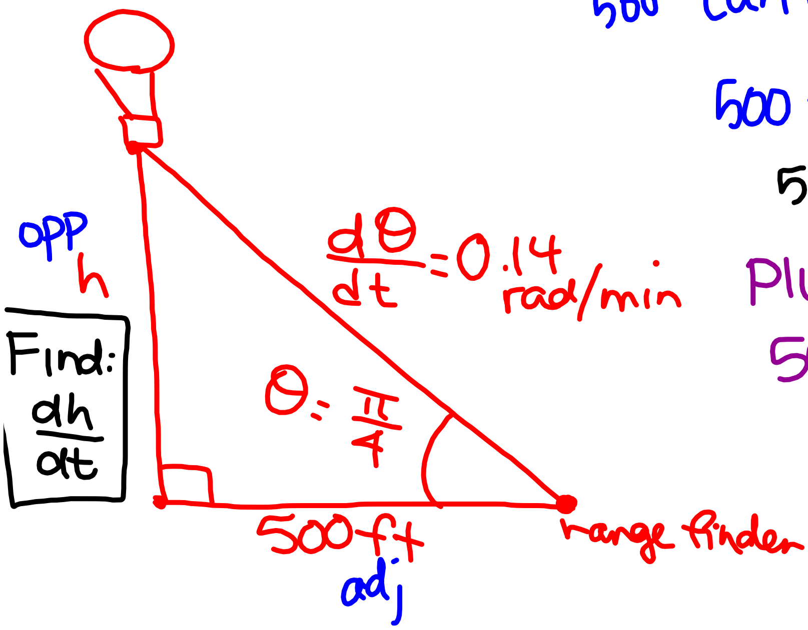
$\frac{\cancel{1.2}\frac{dx}{dt}}{\cancel{1.2}}=\frac{126}{1.2}$

$\boxed{\frac{dx}{dt}=105\text{mph}}$

Dec 14-2:16 PM

Examples

A hot-air balloon rising straight up from a level field is tracked by a range finder 500 ft from the lift-off point. At the moment the range finder's elevation angle is $\frac{\pi}{4}$, the angle is increasing at the rate of 0.14 radians per minute. How fast is the balloon rising at that moment?



$$500 \cdot \tan\theta = \frac{h}{\cancel{500}} \cdot \cancel{500}$$

$$500 \tan\theta = h$$

$$500 \sec^2\theta \cdot \frac{d\theta}{dt} = \frac{dh}{dt}$$

Plug in:

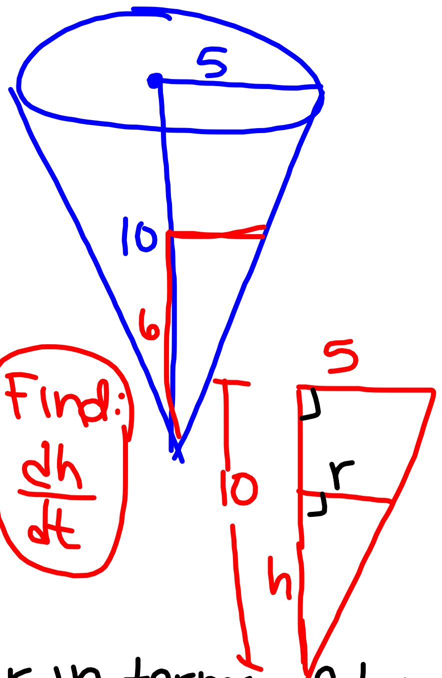
$$500 \sec^2\left(\frac{\pi}{4}\right) \cdot (0.14) = \frac{dh}{dt}$$

$$\boxed{140 = \frac{dh}{dt} \text{ ft/min}}$$

Examples

dV/dt

Water runs into a conical tank at the rate of 9ft³/min. The tank stands point down and has a height of 10ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?



Find: $\frac{dh}{dt}$

r in terms of h:

$$h \cdot \frac{5}{10} = \frac{r}{\cancel{h}} \cdot \cancel{h}$$

$$\boxed{\frac{h}{2} = \frac{1}{2}h = r}$$

$V = \frac{1}{3}\pi r^2 h$

$\left(r = \frac{h}{2}\right)$

r $\quad \cancel{\frac{dr}{dt}}$

h $\quad \frac{dh}{dt}$

V $\quad \frac{dV}{dt}$

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$

$$V = \frac{1}{12}\pi h^3 = \frac{\pi h^3}{12}$$

$$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \frac{dh}{dt}$$

Plug in:

$$9 = \frac{\pi}{12} \cdot 3(6)^2 \cdot \frac{dh}{dt}$$

$$\frac{9}{9\pi} = \frac{\cancel{9\pi} \cdot \frac{dh}{dt}}{\cancel{9\pi}}$$

$$\boxed{0.32 \approx \frac{1}{\pi} = \frac{dh}{dt}}$$
ft/min

Examples

1. A circular pool of water is expanding at the rate of $16\pi \dfrac{\text{in}^2}{\text{sec}}$. At what rate is the radius expanding when the radius is 4 inches?

2. A 25-foot long ladder is leaning against a wall and sliding toward the floor. If the foot of the ladder is sliding away from the base of the wall at a rate of 15 feet per second, how fast is the top of the ladder sliding down the wall when the top of the ladder is 7 feet from the ground?

3. A spherical balloon is expanding at a rate of $60\pi \dfrac{\text{in}^3}{\text{sec}}$. How fast is the surface area of the balloon expanding when the radius of the balloon is 4 in?

$V = \dfrac{4}{3}\pi r^3$ $A = 4\pi r^2$

More Examples

4) An underground conical tank, standing on its vertex, is being filled with water at the rate of $18\pi \frac{\text{ft}^3}{\text{min}}$. If the tank has a height of 30 feet and a radius of 15 feet, how fast is the water level rising when the water is 12 feet deep. (Volume of a cone is $\frac{1}{3}\pi r^2 h$.)

Now try these on your own:

4. Cars *A* and *B* leave town at the same time. Car *A* heads due south at a rate of 80 km/hr, and car *B* heads due west at a rate of 60 km/hr. How fast is the distance between the cars increasing after three hours?

5. A 6-foot-tall woman is walking at the rate of 4 ft/sec away from a street lamp that is 24 feet tall. How fast is the length of her shadow changing?

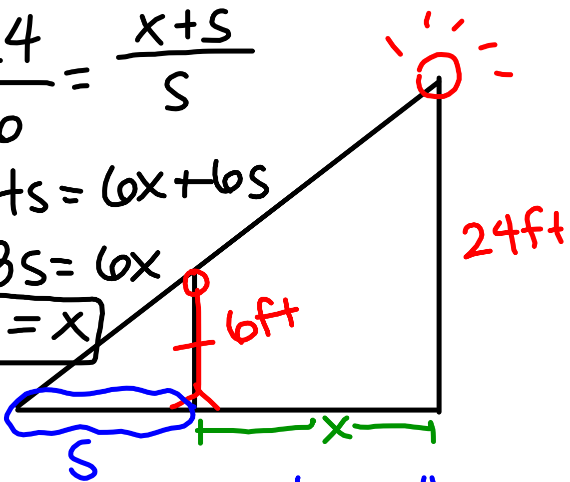
$$\frac{24}{6} = \frac{x+s}{s}$$

$$24s = 6x + 6s$$

$$18s = 6x$$

$$\boxed{3s = x}$$

$$\boxed{s = \frac{x}{3}}$$



$\frac{dx}{dt} = -4$ ft/sec.

Find $\frac{ds}{dt}$

$x =$

$s =$

$$s = \frac{x}{3}$$

$$\frac{ds}{dt} = \frac{1}{3} \cdot \frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{1}{3} \cdot -4$$

$$\frac{ds}{dt} = -\frac{4}{3} \text{ ft/sec}$$

Homework

5.6 pgs.255-258 #3-42 (X3)

Oct 21-8:47 AM