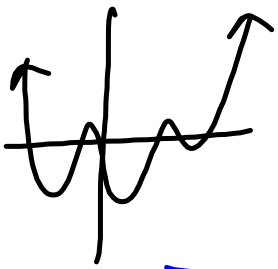
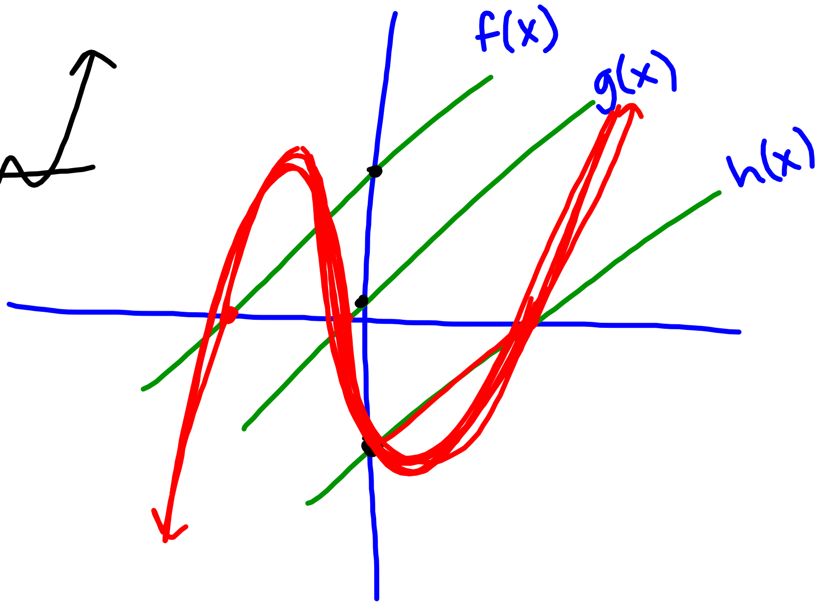
Questions on Lesson 5.5?

We will be taking our content mastery quiz in a few minutes!

$m(x) = f(x) \cdot g(x) \cdot h(x)$





$(x+5)$

$(2x+7)$

$(x-50)$

Feb 27-8:47 AM

5.6 Closing Time
The Closure Property

pg.403-404 in your book

In this chapter you have learned the properties of polynomials in different representations.

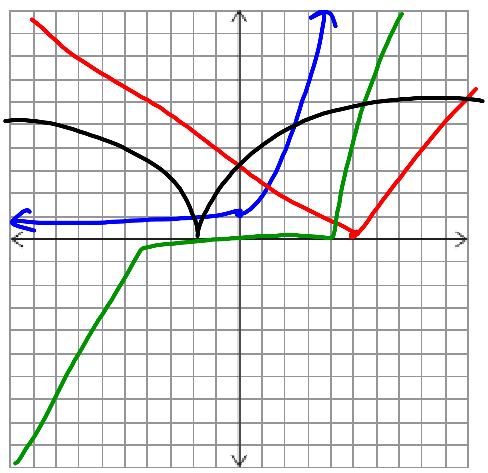
Graphically, polynomials are:	Algebraically, polynomials are:	In a table of values, polynomials are:
• smooth • continuous • increase or decrease to infinity as x approaches positive or negative infinity	• written in the form $ax^n + bx^{n-1} + \cdots$	• made up of real numbers • increase or decrease to infinity as x approaches positive or negative infinity

You have studied many different types of functions. A function has a unique output for every input value. However, a function does not necessarily have to be a polynomial function.

1. Sketch the graphs of two functions that are not polynomial functions. Explain your reasoning.

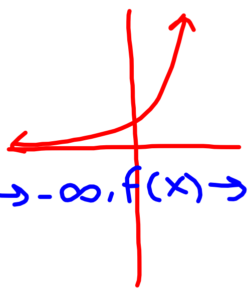
$2x-2 = 2x-2x^0$

a.

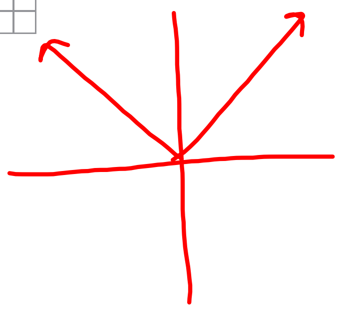


finish #1 on pg.404 and #2 on pg.405 in your book

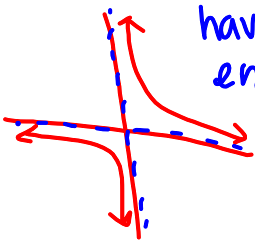
(2)



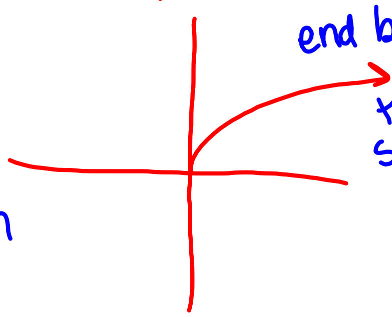
$x \to -\infty, f(x) \to 0$



not cont.
not smooth.



have asymptotes
end beh → 0
not cont
not smooth



end behavior not the same as poly.

pg.405 in your book.

Throughout this chapter you added, subtracted, or multiplied two or more polynomial functions to build a new polynomial function. You did this using a graph, algebra, and a table of values. When an operation is performed on any number or expression in a set and the result is in the same set, it is said to be **closed under that operation.** Are polynomials closed under addition, subtraction, and multiplication? In other words, when you add, subtract, or multiply polynomial functions, will you *always* create another polynomial function?

operation
+ - • ÷

Before answering this question, let's analyze closure within the real number system.

3. Determine whether each set within the Real Number System is closed under addition, subtraction, multiplication, and division.

a. Complete the table. If a set is not closed under a given operation, provide a counterexample.

7 ⋚ 24 | 7+24=31 | 7-24=-17 | 7·24 | 7÷24, $1 \div 3 = 0.\overline{3}$

	Addition	Subtraction	Multiplication	Division
Natural Numbers {1, 2, 3, 4, . . .}	Yes	No 7-24=-17	Yes	No $1 \div 3 = 0.\overline{3}$
Whole Numbers {0, 1, 2, 3, . . .}	Yes	No $2 - 3 = -1$	Yes	No $3 \div 2 = 1.5$
Integers {. . . −2, −1, 0, 1, 2 . . .}	Yes	Yes	Yes	NO $1 \div -2 = -0.5$
Rational Can be represented as the ratio of two integers	Yes	Yes	Yes	NO $\frac{2}{0} = \infty$
Irrational Cannot be represented as the ratio of two integers	Yes	NO	NO	NO

$6\sqrt{2} = \sqrt{2} + 5\sqrt{2}$ $\sqrt{5} - \sqrt{3}$ $\sqrt{2} \cdot \sqrt{2} = 2$
* $\sqrt{5} - \sqrt{5} = 0$ $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$ $\frac{\pi}{\pi} = 1$

b. What patterns do you notice?

Oct 29-9:00 AM

pg.407 in your book.

4. Determine whether polynomial functions are closed under addition, subtraction, multiplication, and division?

a. Write 5 polynomials with various degrees that you will use to explore closure.

$y_1 = 3x^4 - 7x$ $y_2 = -x^8 + \frac{3}{4}x^3 - 1$

$y_3 = -x^3 - 4x^2$

$y_4 = -2x^5 + \frac{1}{2}x + 1$ $y_5 = x^{12} + x^2 - 3$

b. Determine whether the polynomials are closed under addition, subtraction, multiplication, and division. Show all work and explain your reasoning.

CLOSED: + − •

NOT CLOSED: ÷

$$\begin{array}{r} 3x^4 - 7 \\ + \quad -x^3 - 4x^2 \\ \hline 3x^4 - x^3 - 4x^2 - 7 \end{array}$$

finish c and d on pg.407 in your book.

pg.408 in your book.

In the previous problem, *Closed For Business*, you conjectured that integers and polynomials are both closed under addition, subtraction, and multiplication. You also determined through counterexamples that integers and polynomials are not closed under division.

1. Similarities between integer and polynomial operations are shown in the table.

	Integer Example	Polynomial Example
Addition	$\begin{array}{r} 400 + 30 + 7 \\ + \quad 20 + 5 \\ \hline 400 + 50 + 12 \end{array}$	$\begin{array}{r} 4x^2 + 3x + 7 \\ + \quad 2x + 5 \\ \hline 4x^2 + 5x + 12 \end{array}$
Subtraction	$\begin{array}{r} 400 + 30 + 7 \\ - \quad \underline{(20 + 5)} \\ 400 + 10 + 2 \end{array}$	$\begin{array}{r} 4x^2 + 3x + 7 \\ - \quad \underline{(2x + 5)} \\ 4x^2 + x + 2 \end{array}$
Multiplication	$\begin{array}{r} 400 + 30 + 7 \\ \times \quad 20 + 5 \\ \hline 2000 + 150 + 35 \\ 8000 + 600 + 140 \quad \\ \hline 8000 + 2600 + 290 + 35 \end{array}$	$\begin{array}{r} 4x^2 + 3x + 7 \\ \times \quad 2x + 5 \\ \hline 20x^2 + 15x + 35 \\ 8x^3 + 6x^2 + 14x \quad \\ \hline 8x^3 + 26x^2 + 29x + 35 \end{array}$
Division	$\frac{437}{25} = 17\ R12$	$\frac{4x^2 + 3x + 7}{2x + 5} = (2x - 3)\ R(-x + 22)$

a. Describe the similarities between polynomial and integer operations.

b. In what ways is the distributive property essential to performing operations with integers and polynomials?

c. How does this example demonstrate that polynomials are not closed under division?

d. Verify that the polynomial division was performed correctly.

Nov 10-8:45 AM

pg.409 in your book.

You have explored operations under various polynomials. It appears as though polynomials are closed under addition, subtraction, and multiplication, but these examples do not constitute a proof. The real number system is closed, but discovering that polynomials are analogous to the real number system does not allow you to assume that polynomials are also closed. The worked example shows you how to formally prove that polynomials are closed under addition.

Consider the two polynomial functions $f(x)$ and $g(x)$.

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

$g(x) = b_n x^n + b_{n-1} x^{n-1} + \cdots + b_1 x + b_0$

You can show that the polynomials are closed under addition.

Step 1: Write the sum $f(x) + g(x)$. Because the polynomials have multiple terms, it is best to arrange the sum vertically.

$$\begin{array}{rcccccccc} & a_n x^n & + & a_{n-1} x^{n-1} & + \cdots + & a_1 x & + & a_0 \\ + & b_n x^n & + & b_{n-1} x^{n-1} & + \cdots + & b_1 x & + & b_0 \\ \hline \end{array}$$

Step 2: Add the polynomials by combining like terms.

$$\begin{array}{rcccccccc} & a_n x^n & + & a_{n-1} x^{n-1} & + \cdots + & a_1 x & + & a_0 \\ + & b_n x^n & + & b_{n-1} x^{n-1} & + \cdots + & b_1 x & + & b_0 \\ \hline \end{array}$$

$$(a_n + b_n)x^n + (a_{n-1} + b_{n-1})x^{n-1} + \cdots + (a_1 + b_1)x + (a_0 + b_0)$$

Step 3: In the sum, each coefficient is of the form $a_n + b_n$. A coefficient $a_n + b_n$ is a real number because a_n and b_n are real numbers, and the real numbers are closed under addition.

Step 4: The sum of the polynomials $f(x)$ and $g(x)$ is in the form of a polynomial function with a real coefficient. Therefore, polynomials are closed under addition.

finish #2 on pg.410 in your book.

Nov 10-8:46 AM

pg.410 in your book.

2. Consider the two polynomial functions $f(x)$ and $g(x)$.

$$f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$$

$$g(x) = b_nx^n + b_{n-1}x^{n-1} + \cdots + b_1x + b_0$$

a. Prove that polynomials are closed under subtraction.

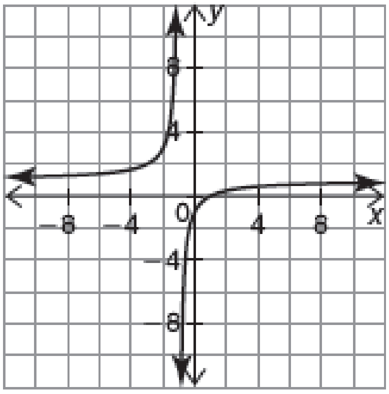
b. Use the multiplication table to prove that polynomials are closed under multiplication.

•	a_nx^n	$a_{n-1}x^{n-1}$	$\cdots$	a_1x	a_0
b_nx^n					
$b_{n-1}x^{n-1}$					
$\vdots$					
b_1x					
b_0					

Nov 10-8:47 AM

Not in your book.

1. Ralph builds a function by performing one of the 4 basic operations (addition, subtraction, multiplication, or division) on 2 polynomial functions. The graph of the resulting function is shown. Which of the 4 basic operations could Ralph have used on the 2 polynomial functions to build his function? Explain your reasoning.

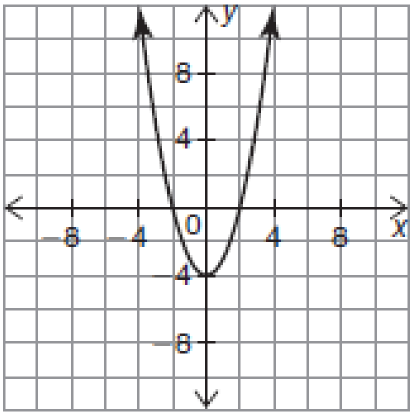


2. Write 2 polynomial functions $f(x)$ and $g(x)$ such that $h(x) = \frac{f(x)}{g(x)}$ and such that $h(x)$ is a polynomial function. Does your solution prove that polynomial functions are closed under division? Explain your reasoning.

Nov 10-8:48 AM

Not in your book.

3. Tamika builds a function by performing one of the 4 basic operations (addition, subtraction, multiplication, or division) on 2 polynomial functions. The graph of the resulting quadratic function is shown. Which of the 4 basic operations could Tamika have used on the 2 polynomial functions to build her function? Explain your reasoning.



4. Let $t(x) = f(x) \cdot g(x) - h(x)$, where $f(x)$ is a quadratic function, $g(x)$ is a cubic function, and $h(x)$ is a quintic function. Is $t(x)$ a polynomial function? Explain your reasoning.

5. Provide a counterexample which proves that polynomial functions are not closed under division. Explain how to verify your answer graphically.

Homework
Finish Lesson 5.6

Oct 21-8:47 AM