Questions on 5.2/5.3 HW?

Nov 3-2:38 PM

5.4 Modeling and Optimization

What does **optimization** mean?

- maximum or minimum
- extreme values

Steps for Solving Max/Min (optimization) problems

1. Understand the Problem Read the problem carefully. Identify the information you need to solve the problem.

2. Develop a Mathematical Model of the Problem Draw pictures and label the parts that are important to the problem. Introduce a variable to represent the quantity to be maximized or minimized. Using that variable, write a function whose extreme value gives the information sought.

3. Graph the function Find the domain of the function. Determine what values of the variable make sense in the problem.

4. Identify the Critical Points and Endpoints Find where the derivative is zero or fails to exist.

5. Solve the Mathematical Model If unsure of the result, support or confirm your solution with another method.

6. Interpret the Solution Translate your mathematical result into the problem setting and decide whether the result makes sense.

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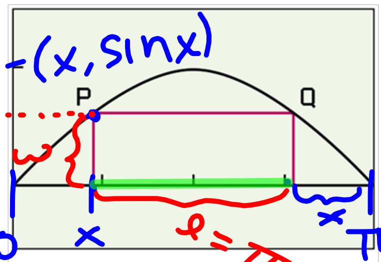
Example

A rectangle is to be inscribed under one arch of the sine curve. What is the largest area the rectangle can have, and what dimensions give that area?

$A = lw$

$(x, \sin x)$

$\sin x = w$



[0, π] by [–0.5, 1.5]

$l = \pi - 2x$

$A(x) = (\pi - 2x)(\sin x)$

$A'(x) = (\pi - 2x)(\cos x) + (\sin x)(-2)$

$A'(x) = (\pi - 2x)(\cos x) - 2\sin x$

$0 = (\pi - 2x)(\cos x) - 2\sin x$

$A'(x) = 0$ when $x = 0.71$

$A(0.71) = (\pi - 2(0.71))(\sin(0.71))$

$\rightarrow A(0.71) = 1.12$

$\pi - 2x > 0$

$\frac{\pi}{2} > \frac{2x}{2}$

$\frac{\pi}{2} > x$

$l = \pi - 2x$

$l = \pi - 2(0.71)$

$\rightarrow l = 1.72$

$w = \sin(0.71)$

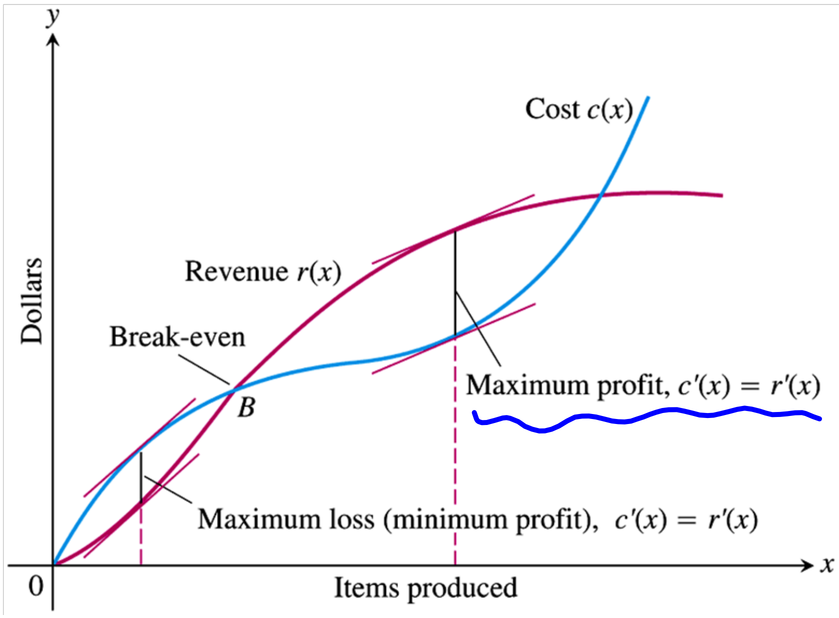
$\rightarrow w \approx 0.65$

Answer

Let $(x, \ \sin x)$ be the coordinates of point P and the x-coordinate of Q is $(\pi - x)$. Thus $\pi - 2x$ = length of rectangle and $\sin x$ = height of rectangle. $A(x) = (\pi - 2x)\sin x$ where $0 \le x \le \pi/2$. Notice that $A(0) = A(\pi/2) = 0$. Find the critical values: $A'(x) = -2\sin x + (\pi - 2x)\cos x$. Let $A'(x) = 0$ and use a graphing calculator to find the solution(s). $A'(x) = 0$ at $x \approx 0.71$. The area of the rectangle is $A(0.71) = 1.12$, where the length is 1.72 and its height is 0.65.

Maximum Profit

Maximim profit (if any) occurs at a production level at which marginal revenue equals marginal cost.



Example

max. profit : $c'(x) = r'(x)$

Suppose that $r(x) = 9x$ and $c(x) = x^3 - 6x^2 + 15x$, where x represents thousands of units. Is there a production level that maximizes profit? If so, what is it?

$r(x) = 9x$

$r'(x) = 9$

$C(x) = x^3 - 6x^2 + 15x$

$c'(x) = 3x^2 - 12x + 15$

$3x^2 - 12x + 15 = 9$

$3x^2 - 12x + 6 = 0$

$3(x^2 - 4x + 2) = 0$

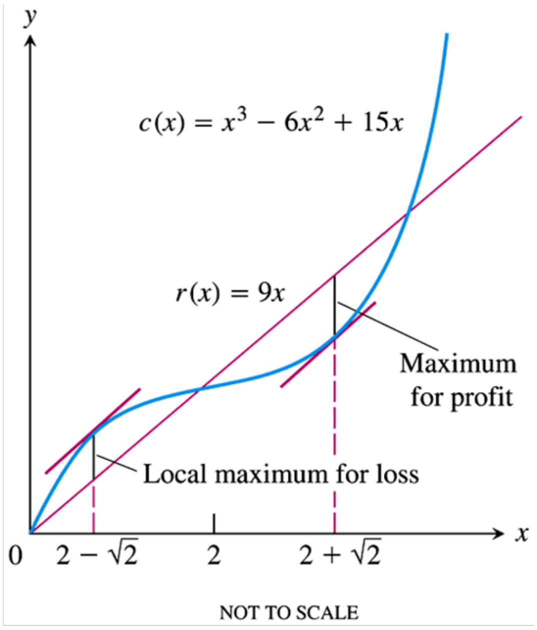
$3(x + 2 + \sqrt{2})(x + 2 - \sqrt{2})$

$x = 2 \pm \sqrt{2}$

max. profit $x + 2 + \sqrt{2} \approx 3.41$

$x + 2 - \sqrt{2} \approx 0.586$

Answer



Set $r'(x) = 9$ equal to $c'(x) = 3x^2 - 12x + 15$.

$3x^2 - 12x + 15 = 9$

$3x^2 - 12x + 6 = 0$

Use the quadratic equation to find

$x_1 = 2 - \sqrt{2} \approx 0.586$

$x_2 = 2 + \sqrt{2} \approx 3.414$

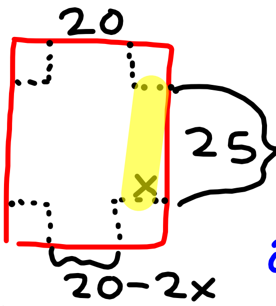
Use a graph to determine that the maximum profit occurs at $x \approx 3.414$.

Minimizing Average Cost

The production level (if any) at which average cost is smallest is a level at which the average cost equals the marginal cost.

Examples

An open-top box is to be made by cutting congruent squares of side length *x* from the corners of a 20- by 25-inch sheet of tin and bending up the sides. How large should the squares be to make the box hold as much as possible? What is the resulting volume?



$25-2x$

$V(x) = x(20-2x)(25-2x)$

$V(x) = x(500-90x+4x^2)$

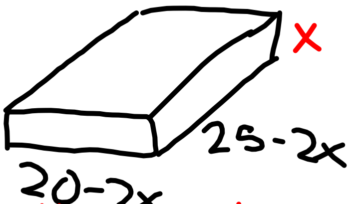
$V(x) = 4x^3 - 90x^2 + 500x$

$20-2x>0$

$20>2x$

$\boxed{10>x}$

$V = l \cdot w \cdot h$



$0<x<10$

$V'(x) = 12x^2 - 180x + 500$

$0 = 12x^2 - 180x + 500$

critical values:

$$c = \frac{-(-180) \pm \sqrt{(-180)^2 - 4 \cdot 12 \cdot 500}}{2 \cdot 12} =$$

$$c = \frac{180 \pm 91.7}{24}$$

$c = \cancel{11.32},\ 3.68$

not in domain

critical value:
$V(3.68) = 820.5$

endpoints
$V(0) = 0$
$V(10) = 0$

maximum
$(3.68, 820.5)$

You have been asked to design a one-liter oil can shaped like a right circular cylinder. What dimensions will use the least material?

Examples

What is the smallest perimeter possible for a rectangle whose area is 16 in^2, and what are its dimensions?

Find two numbers whose sum is 20 and whose product is as large as possible.

Homework

5.4 pgs.230-231 #1-19odds, 20,23,25

Oct 21-8:47 AM