

Questions on Lesson 5.2?

If not, get ready to begin Lesson 5.3!

Function Makeover

Transformations and Symmetry of Polynomial Functions

5.3

PG.348 IN YOUR BOOK

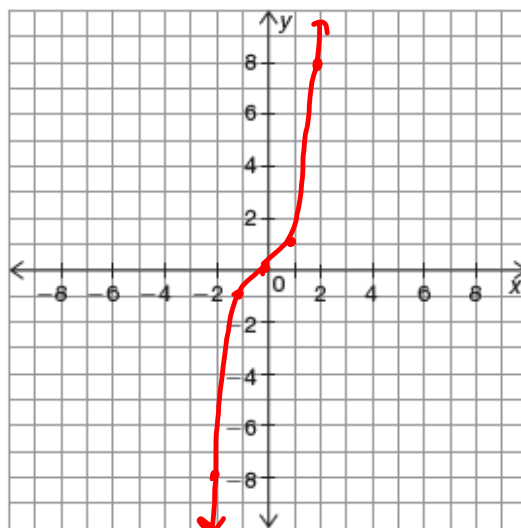
Recall that reference points are a set of points that are used to graph a basic function. Previously, you used reference points and the key characteristics of a parabola to graph the basic quadratic function. You learned that the reference points for the basic quadratic function are $(0, 0)$, $(1, 1)$, and $(2, 4)$. The basic quadratic function is symmetric about the y -axis; that is, $f(x) = f(-x)$. Therefore, you can use symmetry to graph two other points of the basic function, $(-1, 1)$, $(-2, 4)$.

Let's consider a set of reference points and the property of symmetry to graph the basic cubic function.

To complete Questions 1 and 2, consider the basic cubic function, $f(x) = x^3$.

- Complete the table for the given reference points. Then graph the points on the coordinate plane shown.

x	$f(x) = x^3$
0	0
1	1
2	8
-1	-1
-2	-8



The pattern for a basic cubic function is to cube the input value to get the output value. So, from the origin, move over 1 unit and up 1 unit. For the next point, start at the origin, move over 2 units and up 8 units.



- The graph of the basic cubic function is symmetric about the origin. So, $f(x) = -f(-x)$. Use the property of symmetry to determine 2 other points from the reference points. Then, use these points to graph the basic cubic function on the coordinate plane shown.

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Transformations performed on a function $f(x)$ to form a new function $g(x)$ can be described by the transformational function:

$$g(x) = Af(B(x - C)) + D.$$

Previously, you graphed quadratic functions using this notation. You can use this notation to identify the transformations to perform on any function.

Recall that the constants A and D affect the *outside* of the function (the output values). For instance, if $A = 2$, then you can multiply each y -coordinate of $f(x)$ by 2 to determine the y -coordinates of $g(x)$. → y-values

The constants B and C affect the *inside* of the function (the input values). For instance, if $B = 2$, then you can multiply each x -coordinate of $f(x)$ by $\frac{1}{2}$ to determine the x -coordinates of $g(x)$. → x-values

Function Form	Equation Information	Description of Transformation of Graph
$y = Af(x)$	$ A > 1$	vertical stretch of the graph by a factor of A units
	$0 < A < 1$	vertical compression of the graph by a factor of A units
	$A < 0$	reflection across the x -axis
$y = f(Bx)$	$ B > 1$	compressed horizontally by a factor of $\frac{1}{ B }$
	$0 < B < 1$	stretched horizontally by a factor of $\frac{1}{ B }$
	$B < 0$	reflection across the y -axis
$y = f(x - C)$	$C > 0$	horizontal shift right C units
	$C < 0$	horizontal shift left C units
$y = f(x) + D$	$D > 0$	vertical shift up D units
	$D < 0$	vertical shift down D units

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answer #1 with your groups

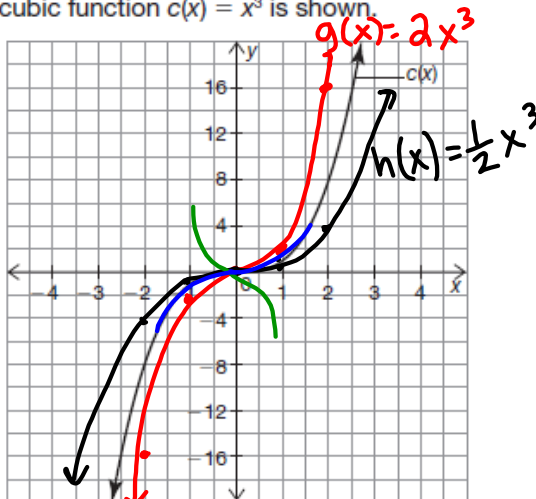
1. Complete the table to show the coordinates of $g(x) = Af(B(x - C)) + D$ after each type of transformation performed on $f(x)$.

$(x+5)^2 - 2$ $(x-5)^2 + 2$

Type of Transformation Performed on $f(x)$	Coordinates of $f(x)$ → Coordinates of $g(x)$
Vertical Dilation by a Factor of A	$(x, y) \rightarrow (\underline{x}, \underline{Ay})$
Horizontal Dilation by a Factor of B	$(x, y) \rightarrow (\underline{\frac{1}{B}x}, \underline{y})$
Horizontal Translation of C units	$(x, y) \rightarrow (\underline{x+C}, \underline{y})$
Vertical Translation of D units	$(x, y) \rightarrow (\underline{x}, \underline{y+D})$
All four transformations: $A, B, C,$ and D	$(x, y) \rightarrow (\underline{\frac{1}{B}x+C}, \underline{Ay+D})$

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2. The graph of the basic cubic function $c(x) = x^3$ is shown.



$A = 2$, so $\cdot y$ by 2

- a. Suppose that $g(x) = 2c(x)$. Use reference points and properties of symmetry to complete the table of values for $g(x)$. Then, graph and label $g(x)$ on the same coordinate plane as $c(x)$.

Reference Points on $c(x)$	→	Corresponding Points on $g(x)$
(0, 0)	→	(0, 0)
(1, 1)	→	(1, 2)
(2, 8)	→	(2, 16)

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answer b-d with your groups

$A = \frac{1}{2}$, so $\cdot y$ by $\frac{1}{2}$

- b. Suppose that $h(x) = \frac{1}{2}c(x)$. Use reference points and properties of symmetry to complete the table of values for $h(x)$. Then, graph and label $h(x)$ on the same coordinate plane as $c(x)$ and $g(x)$.

Reference Points on $c(x)$	→	Corresponding Points on $h(x)$
(0, 0)	→	(0, 0)
(1, 1)	→	(1, $\frac{1}{2}$)
(2, 8)	→	(2, 4)

- c. Describe the symmetry of $g(x)$ and $h(x)$. How does the symmetry of $g(x)$ and $h(x)$ compare to the symmetry of $c(x)$?

Symmetry about the origin.

- d. Determine whether $g(x)$ and $h(x)$ are even functions, odd functions, or neither. Verify your answer algebraically.

$$g(x) = 2x^3 \qquad h(x) = \frac{1}{2}x^3$$

$$g(-x) = 2(-x)^3 \qquad h(-x) = \frac{1}{2}(-x)^3$$

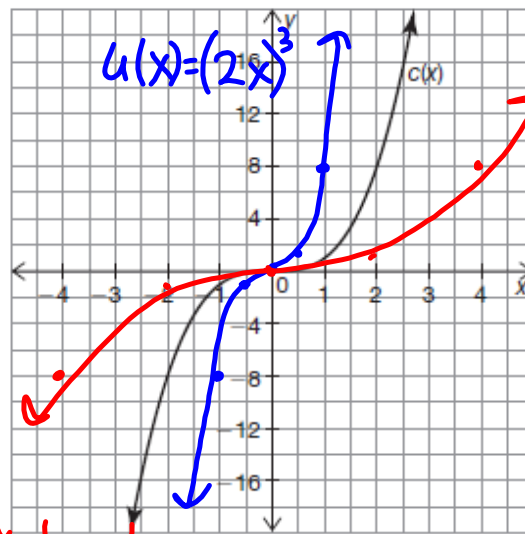
$$g(-x) = -2x^3 \qquad h(-x) = -\frac{1}{2}x^3$$

$$g(-x) = -g(x) \qquad h(-x) = -h(x)$$

SO BOTH ARE ODD FUNCTIONS.

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3. The graph of the basic cubic function $c(x) = x^3$ is shown.



$B = 2$, so $\cdot x$ by $\frac{1}{2}$

- a. Suppose that $u(x) = c(2x)$. Use reference points and properties of symmetry to complete the table of values for $u(x)$. Then, graph and label $u(x)$ on the same coordinate plane as $c(x)$.

Reference Points on $c(x)$	→	Corresponding Points on $u(x)$
(0, 0)	→	(0, 0)
(1, 1)	→	($\frac{1}{2}$, 1)
(2, 8)	→	(1, 8)

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answer b-d with your group

$B = \frac{1}{2}$, so $\cdot x$ by 2

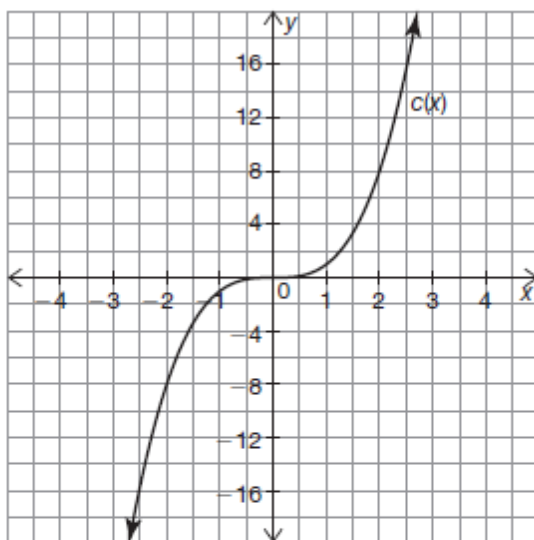
- b. Suppose that $v(x) = c(\frac{1}{2}x)$. Use reference points and properties symmetry to complete the table of values for $v(x)$. Then, graph and label $v(x)$ on the same coordinate plane as $c(x)$ and $u(x)$.

Reference Points on $c(x)$	→	Corresponding Points on $v(x)$
(0, 0)	→	(0, 0)
(1, 1)	→	(2, 1)
(2, 8)	→	(4, 8)

- c. Describe the symmetry of $u(x)$ and $v(x)$. How does the symmetry of $u(x)$ and $v(x)$ compare to the symmetry of $c(x)$?
- d. Determine whether $u(x)$ and $v(x)$ are even functions, odd functions, or neither. Verify your answer algebraically.

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4. The graph of the basic cubic function $c(x) = x^3$ is shown.



$$a(x) = -x^3$$

$A = -1$, so \bullet y by -1

- a. Suppose that $a(x) = -c(x)$. Use reference points and properties of symmetry to complete the table of values for $a(x)$. Then, graph and label $a(x)$ on the same coordinate plane as $c(x)$.

Reference Points on $c(x)$	\rightarrow	Corresponding Points on $a(x)$
(0, 0)	\rightarrow	
(1, 1)	\rightarrow	
(2, 8)	\rightarrow	

$$b(x) = (-x)^3$$

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answer b-d with your group

$B = -1$, so \bullet x by $\frac{1}{-1}$ or -1

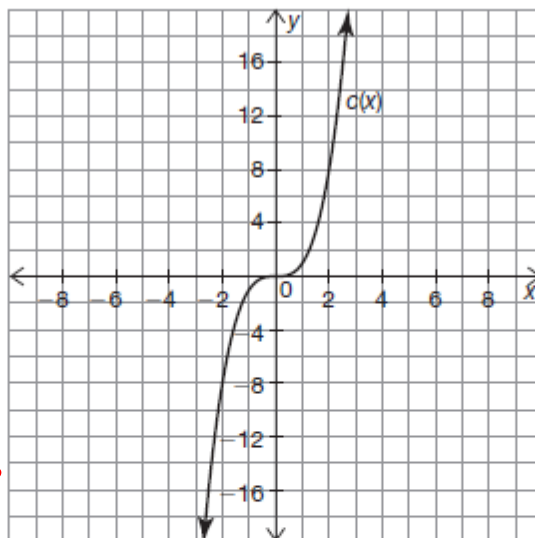
- b. Suppose that $b(x) = c(-x)$. Use reference points and properties symmetry to complete the table of values for $b(x)$. Then, graph and label $b(x)$ on the same coordinate plane as $c(x)$ and $a(x)$.

Reference Points on $c(x)$	\rightarrow	Corresponding Points on $b(x)$
(0, 0)	\rightarrow	
(1, 1)	\rightarrow	
(2, 8)	\rightarrow	

- c. Describe the symmetry of $a(x)$ and $b(x)$. How does the symmetry of $a(x)$ and $b(x)$ compare to the symmetry of $c(x)$?
- d. Determine whether $a(x)$ and $b(x)$ are even functions, odd functions, or neither. Verify your answer algebraically.

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5. The graph of the basic cubic function $c(x) = x^3$ is shown.



$$m(x) = (x-5)^3$$

$C=5$, so $+5$ to x

- a. Suppose that $m(x) = c(x - 5)$. Use reference points and properties of symmetry to complete the table of values for $m(x)$. Then, graph and label $m(x)$ on the same coordinate plane as $c(x)$.

Reference Points on $c(x)$	→	Corresponding Points on $m(x)$
(0, 0)	→	
(1, 1)	→	
(2, 8)	→	

$$n(x) = (x+5)^3$$

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answer b-d with your group

$$C = -5$$
 , so $+ -5$ to x

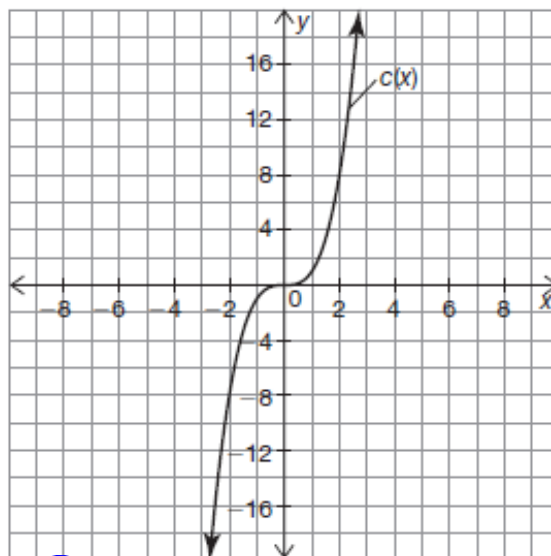
- b. Suppose that $n(x) = c(x + 5)$. Use reference points and properties of symmetry to complete the table of values for $n(x)$. Then, graph and label $n(x)$ on the same coordinate plane as $c(x)$ and $m(x)$.

Reference Points on $c(x)$	→	Corresponding Points on $n(x)$
(0, 0)	→	
(1, 1)	→	
(2, 8)	→	

- c. Describe the symmetry of $m(x)$ and $n(x)$. How does the symmetry of $a(x)$ and $b(x)$ compare to the symmetry of $c(x)$?
- d. Determine whether $m(x)$ and $n(x)$ are even functions, odd functions, or neither. Verify your answer algebraically.

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6. The graph of the basic cubic function $c(x) = x^3$ is shown.



$$j(x) = x^3 + 5$$

$D=5$, so $+5$ to y

- a. Suppose that $j(x) = c(x) + 5$. Use reference points and properties of symmetry to complete the table of values for $j(x)$. Then, graph and label $j(x)$ on the same coordinate plane as $c(x)$.

Reference Points on $c(x)$	→	Corresponding Points on $j(x)$
(0, 0)	→	
(1, 1)	→	
(2, 8)	→	

$$k(x) = x^3 - 5$$

$D=-5$, so $+ -5$ to y

- b. Suppose that $k(x) = c(x) - 5$. Use reference points and properties of symmetry to complete the table of values for $k(x)$. Then, graph and label $k(x)$ on the same coordinate plane as $c(x)$ and $j(x)$.

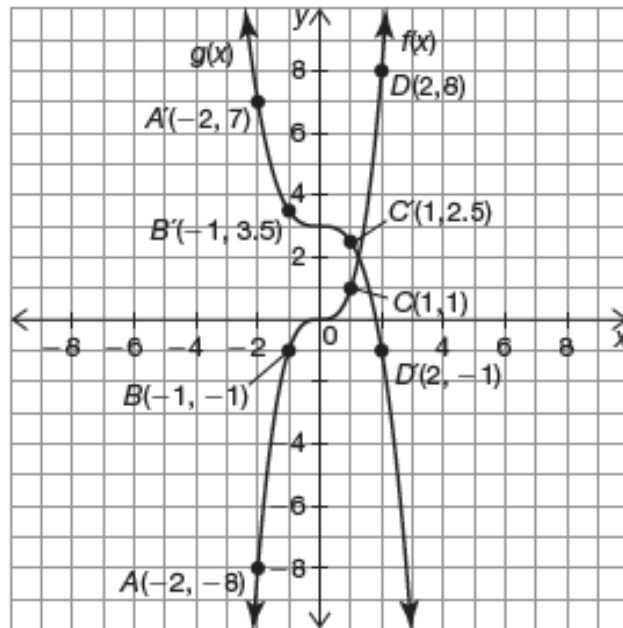
Reference Points on $c(x)$	→	Corresponding Points on $k(x)$
(0, 0)	→	
(1, 1)	→	
(2, 8)	→	

- c. Describe the symmetry of $j(x)$ and $k(x)$. How does the symmetry of $j(x)$ and $k(x)$ compare to the symmetry of $c(x)$?
- d. Determine whether $j(x)$ and $k(x)$ are even functions, odd functions, or neither. Verify your answer algebraically.

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answer b-d with your group

NOT IN YOUR BOOK

1. Analyze the graphs of the functions $f(x)$ and $g(x)$.



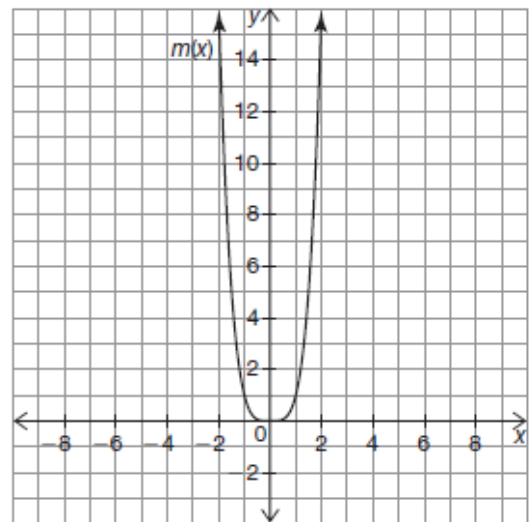
- Write the equation for $f(x)$.
- The function $g(x)$ is a transformation of the function $f(x)$. Describe the transformations performed on $f(x)$ that resulted in the function $g(x)$. Explain your reasoning.
- Write the equation for $g(x)$.
- Is the function $g(x)$ even, odd, or neither? Explain your reasoning.

NOT IN YOUR BOOK

2. The graph of the basic quartic function $m(x) = x^4$ is shown.

a. The function $h(x) = \frac{1}{4}(x - 6)^4$ is a transformation of $m(x)$. Complete the table.

Reference Points on $m(x)$	→	Corresponding Points on $h(x)$
(x, y)	→	
$(-2, 16)$	→	
$(-1, 1)$	→	
$(0, 0)$	→	
$(1, 1)$	→	
$(2, 16)$	→	



b. Graph the function $h(x) = \frac{1}{4}(x - 6)^4$ on the same coordinate plane as $m(x)$.

c. Is the function $h(x)$ even, odd, or neither? Explain your reasoning.

NOT IN YOUR BOOK

3. Consider the polynomial function $p(x) = ax^3 + bx^2 + cx + d$, where a , b , c , and d are real numbers and $a \neq 0$.
- Determine the number of possible zeros for $p(x)$.
 - Determine the number of possible x -intercepts for the graph of $p(x)$.
 - Determine the number of possible maximums and minimums for the graph of $p(x)$.
 - Describe the end behavior of the graph of $p(x)$.

Homework

Finish lesson 5.3