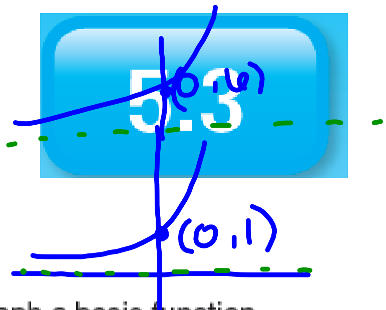
Questions on Lesson 5.2?

If not, get ready to begin Lesson 5.3!

Function Makeover

Transformations and Symmetry of Polynomial Functions

5.3



PG.348 IN YOUR BOOK

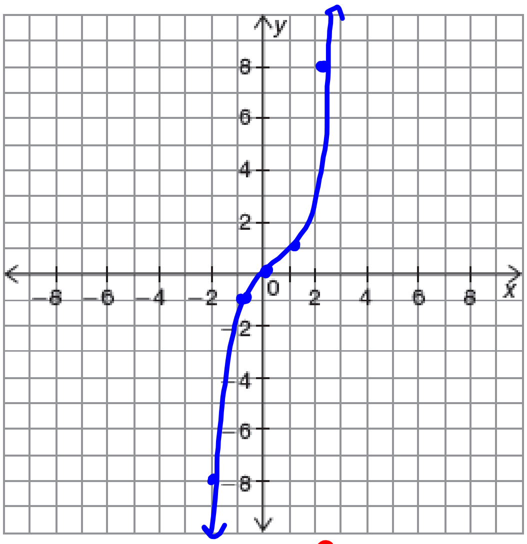
Recall that reference points are a set of points that are used to graph a basic function. Previously, you used reference points and the key characteristics of a parabola to graph the basic quadratic function. You learned that the reference points for the basic quadratic function are (0, 0), (1, 1), and (2, 4). The basic quadratic function is symmetric about the y-axis; that is, $f(x) = f(-x)$. Therefore, you can use symmetry to graph two other points of the basic function, $(-1, 1)$, $(-2, 4)$.

Let's consider a set of reference points and the property of symmetry to graph the basic cubic function.

To complete Questions 1 and 2, consider the basic cubic function, $f(x) = x^3$.

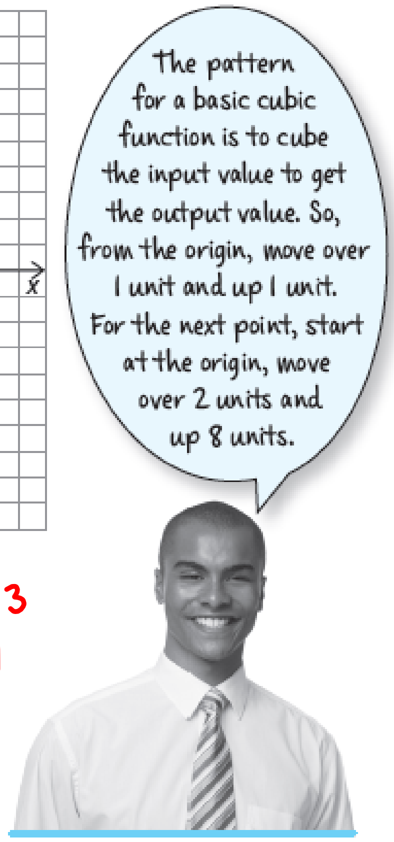
1. Complete the table for the given reference points. Then graph the points on the coordinate plane shown.

x	$f(x) = x^3$
0	0
1	1
2	8
-1	-1
-2	-8



$x^3 + 1$

$(x - 1)^3$



2. The graph of the basic cubic function is symmetric about the origin. So, $f(x) = -f(-x)$. Use the property of symmetry to determine 2 other points from the reference points. Then, use these points to graph the basic cubic function on the coordinate plane shown.

Oct 9-8:12 AM

PG.349 IN YOUR BOOK

Transformations performed on a function $f(x)$ to form a new function $g(x)$ can be described by the transformational function:

$$g(x) = Af(B(x - C)) + D.$$

Previously, you graphed quadratic functions using this notation. You can use this notation to identify the transformations to perform on any function.

y-values

Recall that the constants A and D affect the *outside* of the function (the output values). For instance, if $A = 2$, then you can multiply each y-coordinate of $f(x)$ by 2 to determine the y-coordinates of $g(x)$.

x-values

The constants B and C affect the inside of the function (the input values). For instance, if $B = 2$, then you can multiply each x-coordinate of $f(x)$ by $\frac{1}{2}$ to determine the x-coordinates of $g(x)$.

Function Form	Equation Information	Description of Transformation of Graph
$y = Af(x)$	$\lvert A\rvert > 1$	vertical stretch of the graph by a factor of A units
	$0 < \lvert A\rvert < 1$	vertical compression of the graph by a factor of A units
	$A < 0$	reflection across the x-axis
$y = f(Bx)$	$\lvert B\rvert > 1$	compressed horizontally by a factor of $\frac{1}{\lvert B\rvert}$
	$0 < \lvert B\rvert < 1$	stretched horizontally by a factor of $\frac{1}{\lvert B\rvert}$
	$B < 0$	reflection across the y-axis
$y = f(x - C)$	$C > 0$	horizontal shift right C units
	$C < 0$	horizontal shift left C units
$y = f(x) + D$	$D > 0$	vertical shift up D units
	$D < 0$	vertical shift down D units

PG.350 IN YOUR BOOK
answer #1 with your groups

1. Complete the table to show the coordinates of $g(x) = Af(B(x - C)) + D$ after each type of transformation performed on $f(x)$.

Type of Transformation Performed on $f(x)$	Coordinates of $f(x)$ → Coordinates of $g(x)$
Vertical Dilation by a Factor of A	$(x, y) \rightarrow (x, Ay)$
Horizontal Dilation by a Factor of B	$(x, y) \rightarrow (\frac{1}{B}x, y)$
Horizontal Translation of C units	$(x, y) \rightarrow (x + C, y)$
Vertical Translation of D units	$(x, y) \rightarrow (x, y + D)$
All four transformations: A, B, C, and D	$(x, y) \rightarrow (\frac{1}{B}x + C, Ay + D)$

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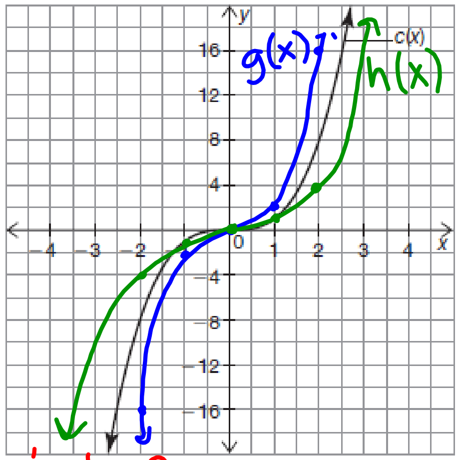
PG.351 IN YOUR BOOK

2. The graph of the basic cubic function $c(x) = x^3$ is shown.



A=2, so • y's by 2

a. Suppose that $g(x) = 2c(x)$. Use reference points and properties of symmetry to complete the table of values for $g(x)$. Then, graph and label $g(x)$ on the same coordinate plane as $c(x)$.

Reference Points on $c(x)$	→	Corresponding Points on $g(x)$
(0, 0)	→	(0, 0)
(1, 1)	→	(1, 2)
(2, 8)	→	(2, 16)

pg.351 in your book
answer b-d with your groups

b. Suppose that $h(x) = \frac{1}{2}c(x)$. Use reference points and properties of symmetry to complete the table of values for $h(x)$. Then, graph and label $h(x)$ on the same coordinate plane as $c(x)$ and $g(x)$.

Reference Points on $c(x)$	→	Corresponding Points on $h(x)$
(0, 0)	→	(0, 0)
(1, 1)	→	$(1, \frac{1}{2})$
(2, 8)	→	(2, 4)

c. Describe the symmetry of $g(x)$ and $h(x)$. How does the symmetry of $g(x)$ and $h(x)$ compare to the symmetry of $c(x)$?

d. Determine whether $g(x)$ and $h(x)$ are even functions, odd functions, or neither. Verify your answer algebraically.

$g(x) = 2x^3$

$g(-x) = 2(-x)^3$

$g(-x) = -2x^3$

$h(x) = \frac{1}{2}x^3$

$h(-x) = \frac{1}{2}(-x)^3$

$h(-x) = -\frac{1}{2}x^3$

$h(-x) = -h(x)$

ODD

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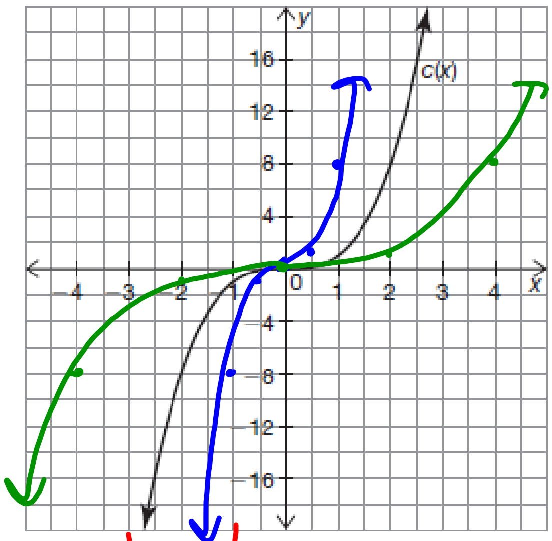
PG.352 IN YOUR BOOK

3. The graph of the basic cubic function $c(x) = x^3$ is shown.



B=2, so •x by $\frac{1}{2}$

a. Suppose that $u(x) = c(2x)$. Use reference points and properties of symmetry to complete the table of values for $u(x)$. Then, graph and label $u(x)$ on the same coordinate plane as $c(x)$.

Reference Points on $c(x)$	→	Corresponding Points on $u(x)$
(0, 0)	→	(0, 0)
(1, 1)	→	($\frac{1}{2}$, 1)
(2, 8)	→	(1, 8)

pg.352 in your book
answer b-d with your group

B=$\frac{1}{2}$, so •x by 2

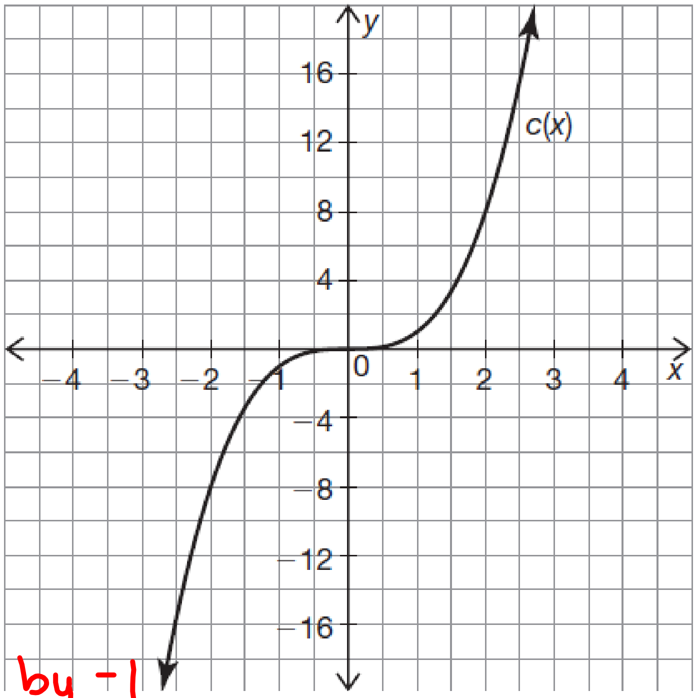
b. Suppose that $v(x) = c\left(\frac{1}{2}x\right)$. Use reference points and properties symmetry to complete the table of values for $v(x)$. Then, graph and label $v(x)$ on the same coordinate plane as $c(x)$ and $u(x)$.

Reference Points on $c(x)$	→	Corresponding Points on $v(x)$
(0, 0)	→	(0, 0)
(1, 1)	→	(2, 1)
(2, 8)	→	(4, 8)

c. Describe the symmetry of $u(x)$ and $v(x)$. How does the symmetry of $u(x)$ and $v(x)$ compare to the symmetry of $c(x)$?

d. Determine whether $u(x)$ and $v(x)$ are even functions, odd functions, or neither. Verify your answer algebraically.

4. The graph of the basic cubic function $c(x) = x^3$ is shown.



A = −1, so •y by −1

a. Suppose that $a(x) = -c(x)$. Use reference points and properties of symmetry to complete the table of values for $a(x)$. Then, graph and label $a(x)$ on the same coordinate plane as $c(x)$.

reflection across x-axis

Reference Points on $c(x)$	→	Corresponding Points on $a(x)$
(0, 0)	→	(0, 0)
(1, 1)	→	(1, −1)
(2, 8)	→	(2, −8)

B = −1, so •x by $\frac{1}{-1}$

b. Suppose that $b(x) = c(-x)$. Use reference points and properties symmetry to complete the table of values for $b(x)$. Then, graph and label $b(x)$ on the same coordinate plane as $c(x)$ and $a(x)$.

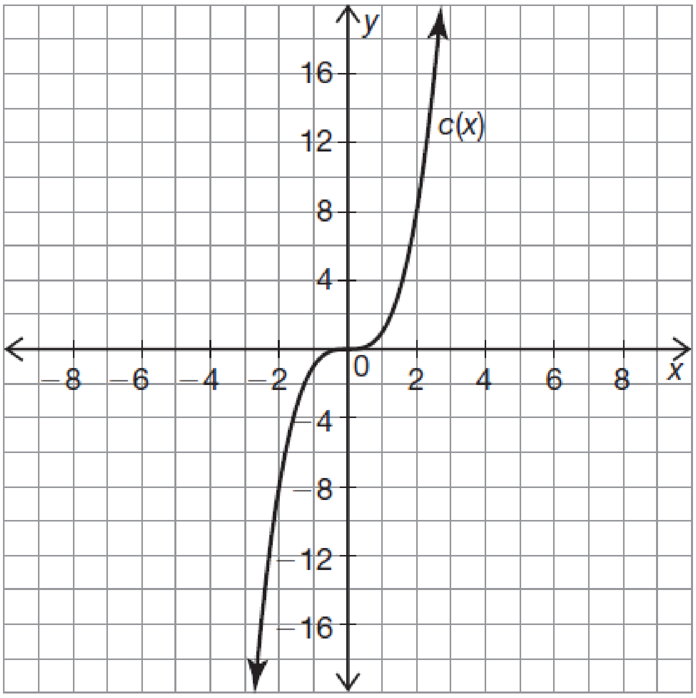
reflection across y-axis

Reference Points on $c(x)$	→	Corresponding Points on $b(x)$
(0, 0)	→	(0, 0)
(1, 1)	→	(1, −1) or (−1, 1)
(2, 8)	→	(2, −8) or (−2, 8)

c. Describe the symmetry of $a(x)$ and $b(x)$. How does the symmetry of $a(x)$ and $b(x)$ compare to the symmetry of $c(x)$?

d. Determine whether $a(x)$ and $b(x)$ are even functions, odd functions, or neither. Verify your answer algebraically.

5. The graph of the basic cubic function $c(x) = x^3$ is shown.



a. Suppose that $m(x) = c(x - 5)$. Use reference points and properties of symmetry to complete the table of values for $m(x)$. Then, graph and label $m(x)$ on the same coordinate plane as $c(x)$.

Reference Points on $c(x)$	→	Corresponding Points on $m(x)$
(0, 0)	→	5, 0
(1, 1)	→	6, 1
(2, 8)	→	7, 8

b. Suppose that $n(x) = c(x + 5)$. Use reference points and properties of symmetry to complete the table of values for $n(x)$. Then, graph and label $n(x)$ on the same coordinate plane as $c(x)$ and $m(x)$.

Reference Points on $c(x)$	→	Corresponding Points on $n(x)$
(0, 0)	→	−5, 0
(1, 1)	→	−4, 1
(2, 8)	→	−3, 8

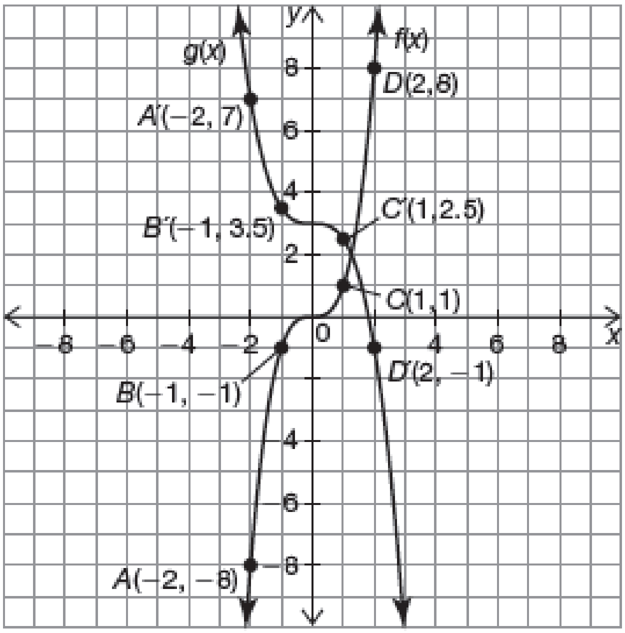
c. Describe the symmetry of $m(x)$ and $n(x)$. How does the symmetry of $a(x)$ and $b(x)$ compare to the symmetry of $c(x)$?

d. Determine whether $m(x)$ and $n(x)$ are even functions, odd functions, or neither. Verify your answer algebraically.

$m(x) = (x-5)^3$ $n(x) = (x+5)^3$

NOT IN YOUR BOOK

1. Analyze the graphs of the functions $f(x)$ and $g(x)$.



a. Write the equation for $f(x)$.

b. The function $g(x)$ is a transformation of the function $f(x)$. Describe the transformations performed on $f(x)$ that resulted in the function $g(x)$. Explain your reasoning.

c. Write the equation for $g(x)$.

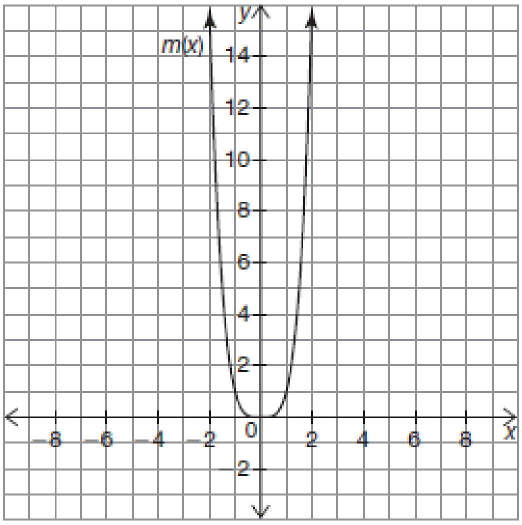
d. Is the function $g(x)$ even, odd, or neither? Explain your reasoning.

NOT IN YOUR BOOK

2. The graph of the basic quartic function $m(x) = x^4$ is shown.

a. The function $h(x) = \frac{1}{4}(x - 6)^4$ is a transformation of $m(x)$. Complete the table.

Reference Points on $m(x)$	$\rightarrow$	**Corresponding Points on $h(x)$**
(x, y)	$\rightarrow$	
$(-2, 16)$	$\rightarrow$	
$(-1, 1)$	$\rightarrow$	
$(0, 0)$	$\rightarrow$	
$(1, 1)$	$\rightarrow$	
$(2, 16)$	$\rightarrow$	



b. Graph the function $h(x) = \frac{1}{4}(x - 6)^4$ on the same coordinate plane as $m(x)$.

c. Is the function $h(x)$ even, odd, or neither? Explain your reasoning.

NOT IN YOUR BOOK

3. Consider the polynomial function $p(x) = ax^3 + bx^2 + cx + d$, where a, b, c, and d are real numbers and $a \neq 0$.

   a. Determine the number of possible zeros for $p(x)$.

   b. Determine the number of possible x-intercepts for the graph of $p(x)$.

   c. Determine the number of possible maximums and minimums for the graph of $p(x)$.

   d. Describe the end behavior of the graph of $p(x)$.

Oct 21-8:50 AM

Homework

Finish lesson 5.3