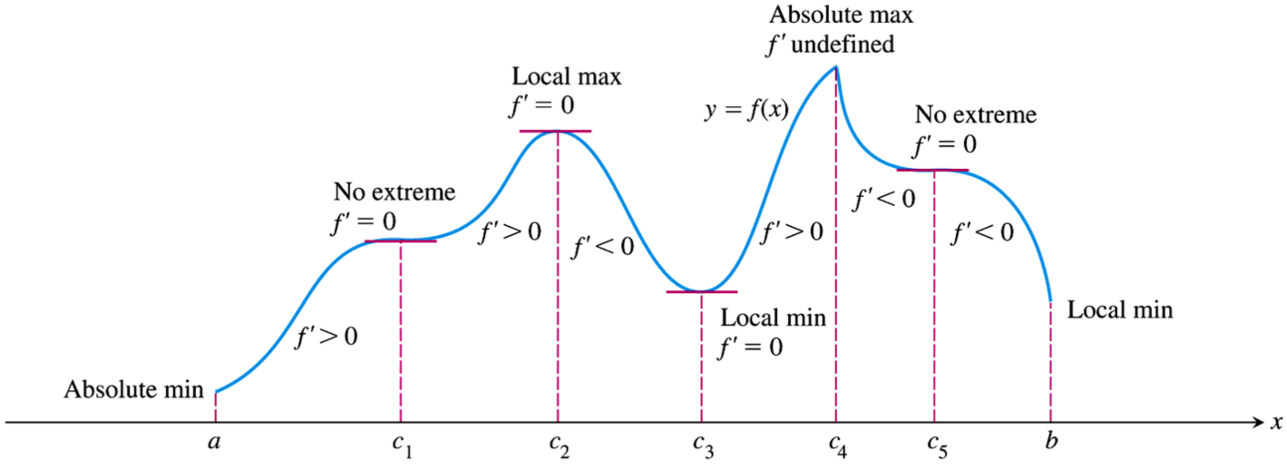
Questions on 5.1/5.2 HW?

5.3 Connecting f' and f" with the Graph of f

First Derivative Test for Local Extrema



pics on pg. 209-210

The following test applies to a continuous function f(x).

At a critical point c :

1. If f' changes sign from positive to negative at c, then f has a local maximum value at c.
2. If f' changes sign from negative to positive at c, then f has a local minimum value at c.
3. If f' does not change sign at c, then f has no local extreme value at c.

At a left endpoint a :

If $f' < 0$ ($f' > 0$) for $x > a$, then f has a local maximum (minimum) value at a.

At a right endpoint b :

If $f' < 0$ ($f' > 0$) for $x < b$, then f has a local minimum (maximum) value at b.

Example

Use the First Derivative Test to find the local extreme values. Identify any absolute extrema. $f(x) = x^3 - 27x + 3$

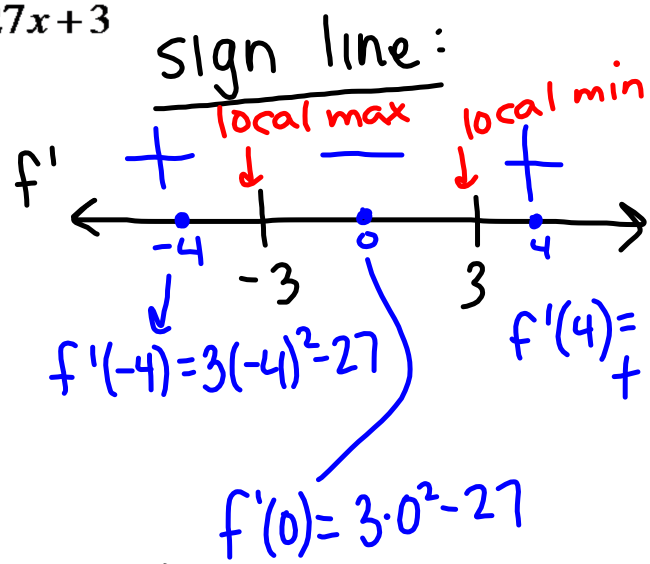
$f'(x) = 3x^2 - 27$

$0 = 3x^2 - 27$

$0 = 3(x^2 - 9)$

$0 = 3(x+3)(x-3)$

$x = 3, -3$



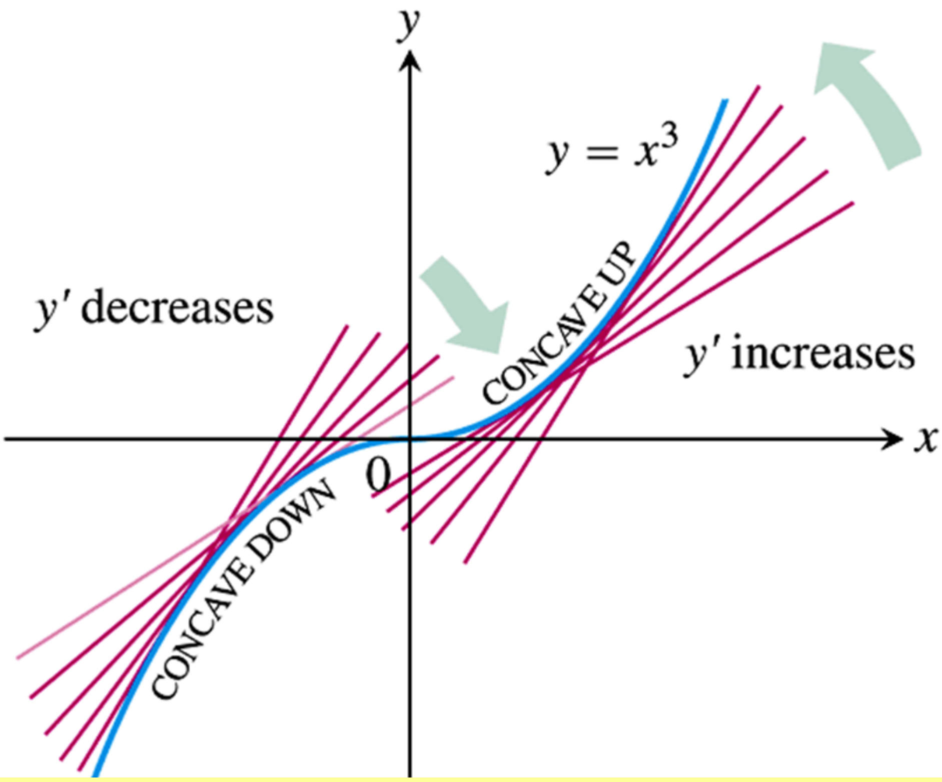
local max: $f(-3) = 57 \rightarrow (-3, 57)$

local min: $f(3) = 51 \rightarrow (3, 51)$

Answer

Since f is differentiable for all real numbers, the only critical points are the zeros of f'. Solving $f'(x) = 3x^2 - 27 = 0$, we find the zeros to be $x = -3$, and $x = 3$. The zeros partition the x-axis into three intervals. Use a sign chart to find the sign on each interval. The First Derivative Test and the sign of f' tells us that there is a local maximum at $x = -3$ and a local minimum at $x = 3$. The local maximum value is $f(-3) = 57$ and the local mimimum value is $f(3) = -51$. The range of $f(x)$ is $(-\infty, \infty)$ so there is no absolute extrema.

Concavity



The graph of a differentiable function $y = f(x)$ is

(a) concave up on an open interval I if y' is increasing on I.

(b) concave down on an open interval I if y' is decreasing on I.

★Concavity Test★

The graph of a twice-differentiable function $y = f(x)$ is

(a) concave up on an open interval where $y'' > 0$.

(b) concave down on an open interval where $y'' < 0$.

Example

Use the Concavity Test to determine the concavity of $f(x) = x^2$ on the interval (2,8).

$f(x) = x^2$

$f'(x) = 2x$

$f''(x) = 2$

→ Since $y'' = 2$ is always positive, the graph of $f(x) = x^2$ is concave up on any interval, in particular, on (2,8).

Answer

Since $y'' = 2$ is always positive, the graph of $y = x^2$ is concave up on any interval. In particular, it is concave up on (2,8).

✯Point of Inflection

A point where the graph of a function has a tangent line and where the concavity changes is a **point of inflection.**

Example

Find all points of inflection of the graph of $y = 2e^{-x^2}$.

Answer

Find the second derivative of $y = 2e^{-x^2}$.

$$y' = 2e^{-x^2}(-2x) = -4xe^{-x^2}$$

$$y'' = -4e^{-x^2} + (-4x)e^{-x^2}(-2x)$$

$$= -4e^{-x^2} + (8x^2)e^{-x^2}$$

$$= 4e^{-x^2}(-1 + 2x^2)$$

The factor $4e^{-x^2}$ is always positive. The factor $(-1+2x^2)$ changes sign at $x = \pm\sqrt{\frac{1}{2}}$. The points of inflection are $\left(-\sqrt{\frac{1}{2}}, \frac{2}{\sqrt{e}}\right)$ and $\left(\sqrt{\frac{1}{2}}, \frac{2}{\sqrt{e}}\right)$.

Second Derivative Test for Local Extrema

1. If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.
2. If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.

Example

Find the local extreme values of $f(x) = x^3 - 6x + 5$.

Answer

$$f'(x) = 3x^2 - 6$$

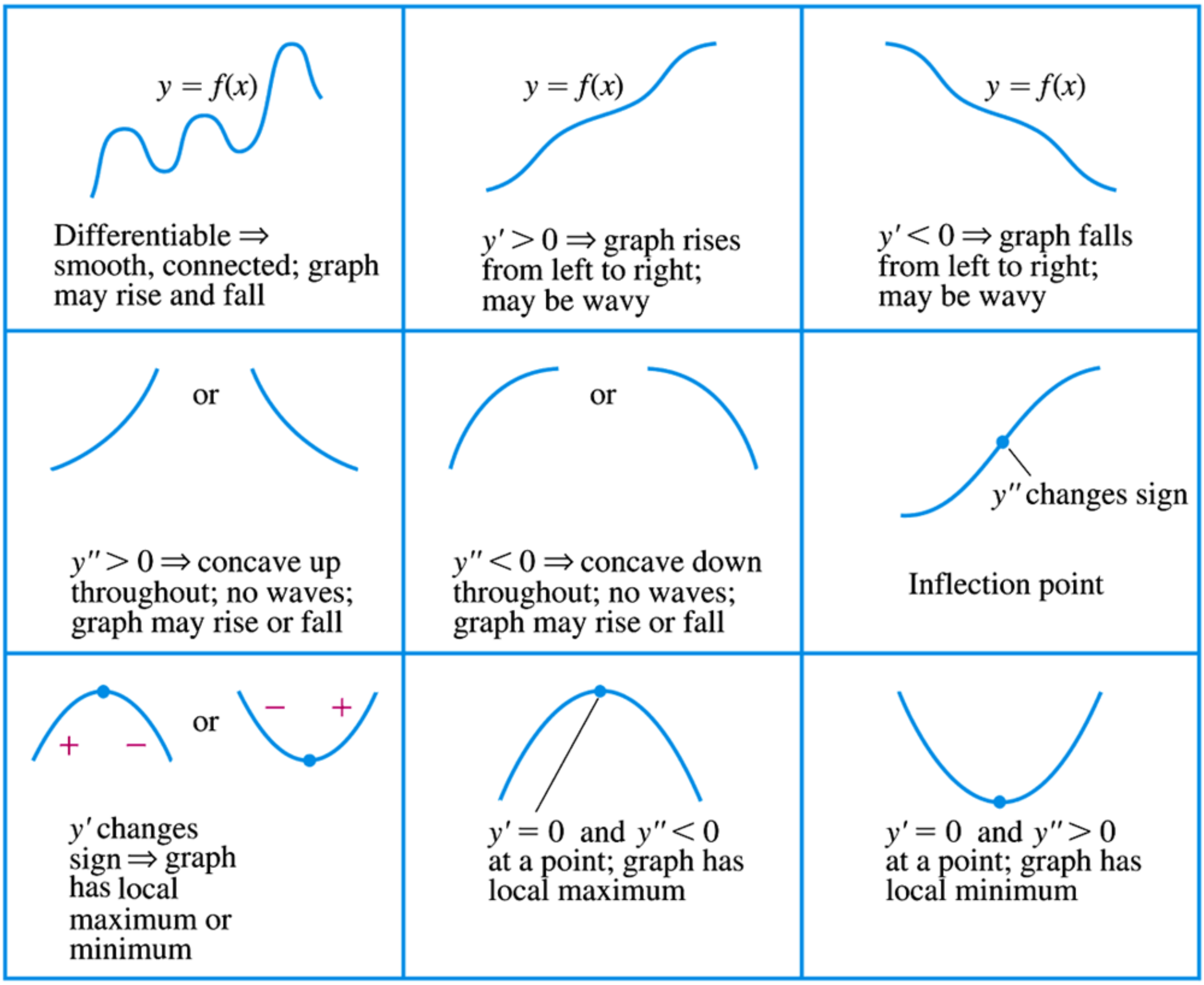
$$f''(x) = 6x.$$

Test the critical points $x = \pm\sqrt{2}$.

$f''\left(-\sqrt{2}\right) = -6\sqrt{2} < 0 \quad \Rightarrow f$ has a local maximum at $x = -\sqrt{2}$ and

$f''\left(\sqrt{2}\right) = 6\sqrt{2} > 0 \quad \Rightarrow f$ has a local minimum at $x = \sqrt{2}$.

Learning about Functions from Derivatives

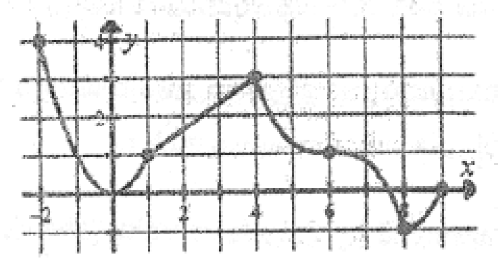


Examples

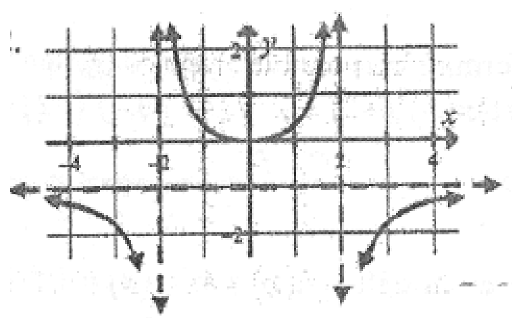
Use the graphs below to answer each of the following questions:

1. On which intervals is the function concave up?
2. On which intervals is the function concave down?
3. On which intervals does the function have no concavity?
4. What are the points of inflection of the function?

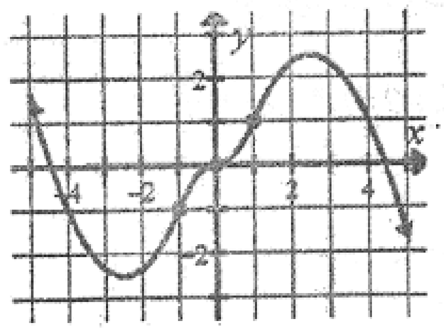
A.



B.



C.



Dec 7-8:45 AM

Homework

5.3 pgs.219-220 #1-11odd, 15-30 (X3)

Oct 21-8:47 AM