Questions on Lesson 5.1?

If not, get ready to begin Lesson 5.2!

$x(x-3)^2(x-1)(x+2)^2(x-7)^6 = 0$

12th degree poly.

Roots: $x = 0, 3, 1, -2, 7$

m: 1 2 1 2 6

5.2 Polynomial Power
Power Functions

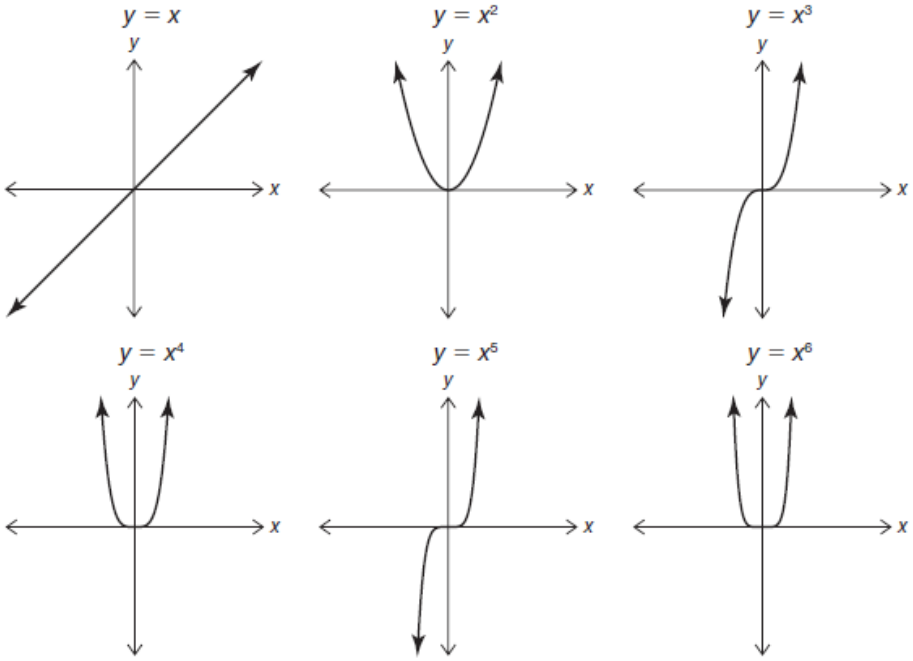
PG.334 IN YOUR BOOK

PROBLEM 1 What Odd Behavior . . . or Is It Even?

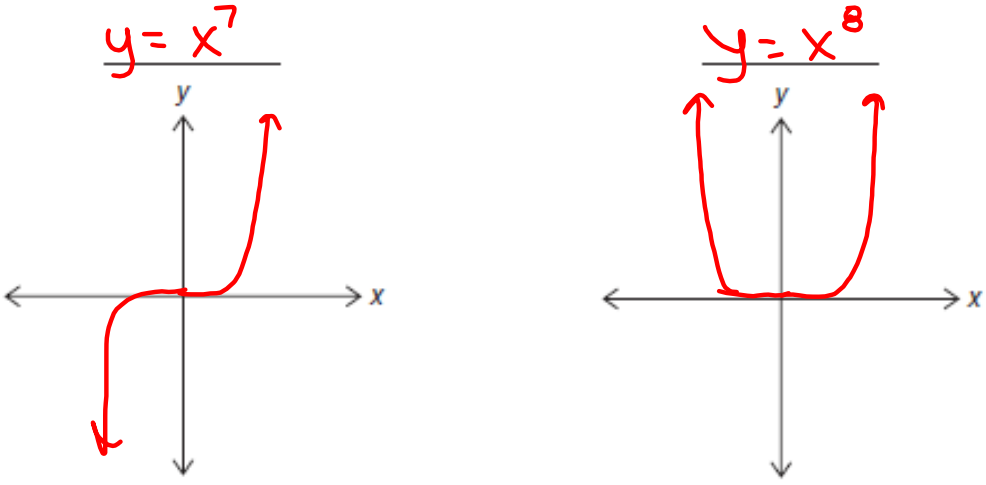
You have studied linear functions, quadratic functions, and now you will explore more polynomial functions. A common type of polynomial function is a *power function*. A **power function** is a function of the form $P(x) = ax^n$, where n is a non-negative integer.

For the purpose of this lesson, you will only focus on power functions where $a = 1$ and -1. In the next lesson you will investigate power functions with various a-values.

1. Consider each power function and its graph in the sequence shown.



a. Sketch and label the next two graphs in the sequence.



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PG.335 IN YOUR BOOK
answer b-d with your groups

b. State any observations or patterns that you notice about the graphs in the sequence.

c. Make a general statement about the graph of a power function raised to an odd degree.

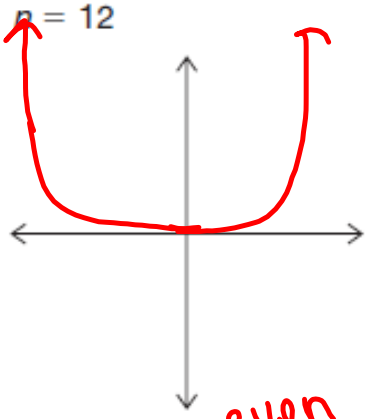
two $\frac{1}{2}$ of a parabola, S-like shape; one end points ↑ one points ↓

d. Make a general statement about the graph of a power function raised to an even degree.

parabola-like shape - both ends pointing ↑ or ↓

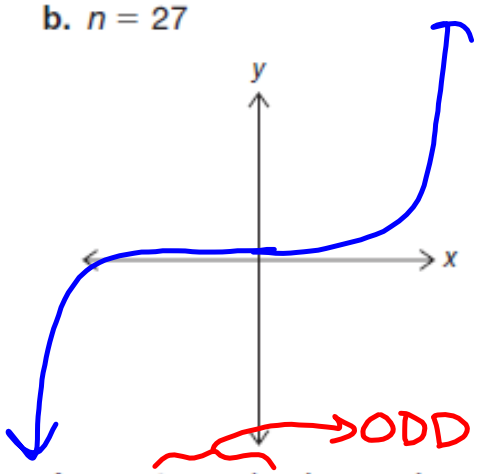
2. Based on you work in Question 1, sketch the graph of x^n when:

a. $n = 12$



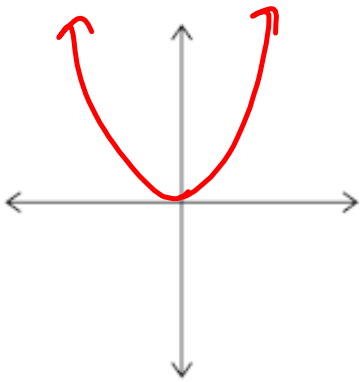
even

b. $n = 27$

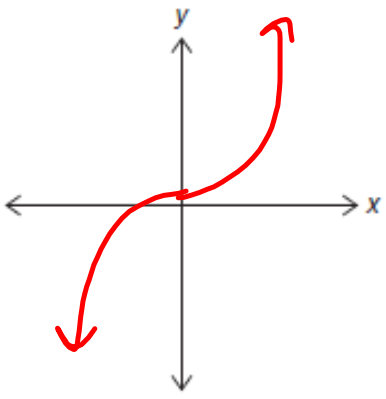


ODD

c. $n = 2m$, where m is an integer greater than 0



d. $n = 2m + 1$, where m is an integer greater than 0



PG.336 IN YOUR BOOK

The **end behavior** of a graph of a function is the behavior of the graph as x approaches infinity and as x approaches negative infinity.

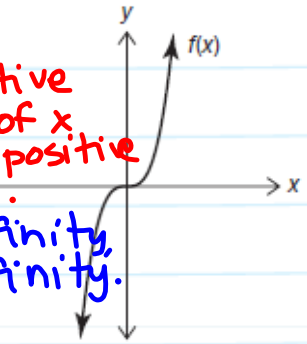
You can write the end behavior of this polynomial function using the notation:

As $x \to \infty$, $f(x) \to \infty$.

As $x \to -\infty$, $f(x) \to -\infty$.

As x approaches positive infinity, f of x approaches positive infinity.

As x approaches negative infinity, f of x approaches negative infinity.

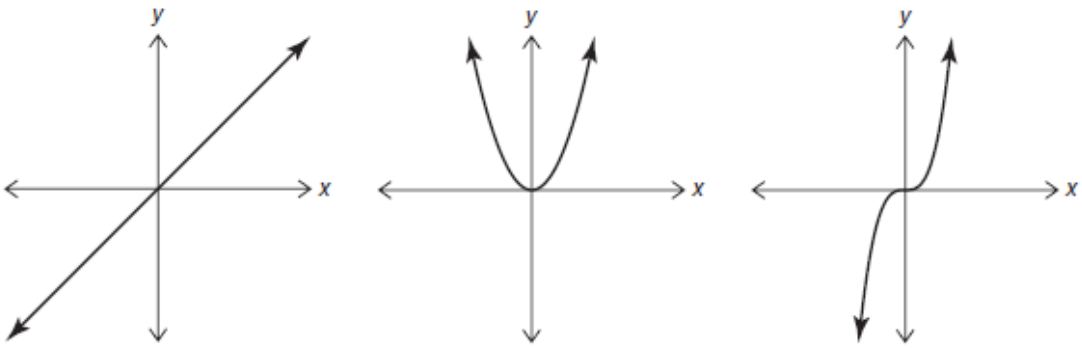


answer 3-4 with your groups

3. Explain in words what the end behavior in the worked example means.

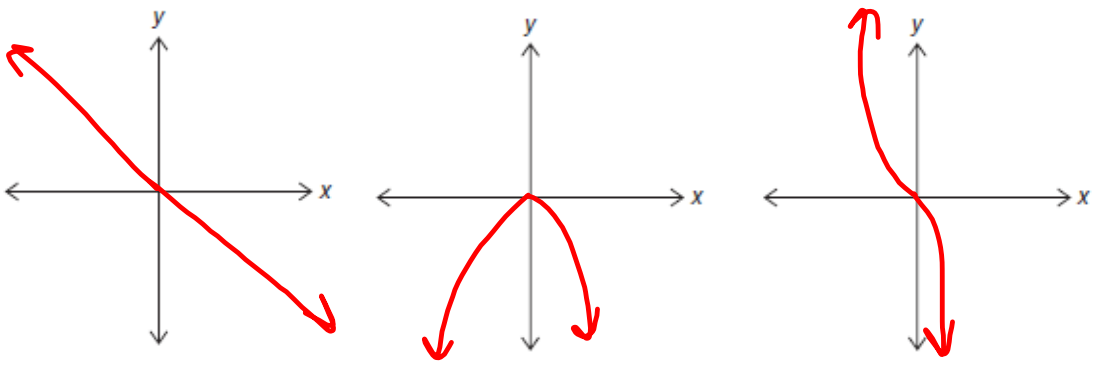
4. Consider the sequence of graphs shown.

$f_1(x) = x$ $f_2(x) = x^2$ $f_3(x) = x^3$



a. Write each function in terms of x, and then sketch it.

$-f_1(x) =$ $-x$ $-f_2(x) =$ $-x^2$ $-f_3(x) =$ $-x^3$



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PG.337 IN YOUR BOOK

$f(x) = ax^3 + bx^2 + cx + d$

b. Complete the table to describe the end behavior for any polynomial function.

	Odd Degree Power Function ($x^1, x^3, x^5, \ldots$)	Even Degree Power Function ($x^2, x^4, x^6, \ldots$)
$a > 0$	As $x \to \infty, f(x) \to \infty$ As $x \to -\infty, f(x) \to -\infty$	As $x \to \infty, f(x) \to \infty$ As $x \to -\infty, f(x) \to \infty$
$a < 0$	As $x \to \infty, f(x) \to -\infty$ As $x \to -\infty, f(x) \to \infty$	As $x \to \infty, f(x) \to -\infty$ As $x \to -\infty, f(x) \to -\infty$

PG.342 IN YOUR BOOK

If a graph is **symmetric about a line**, the line divides the graph into two identical parts. Special attention is given to the line of symmetry when it is the *y*-axis as it tells you that the function is even.

PG.343 IN YOUR BOOK

The graph of an odd degree basic power function is *symmetric about a point*, in particular the origin. A function is **symmetric about a point** if each point on the graph has a point the same distance from the central point, but in the opposite direction. Special attention is given when the central point is the origin as it determines that the function is odd. When the point of symmetry is the origin, the graph is reflected across the *x*-axis and the *y*-axis. If you replace both (x, y) with $(-x, -y)$, the function remains the same.

ALGEBRAIC PROOF:

An **even function** has a graph symmetric about the *y*-axis, thus $f(x) = f(-x)$.

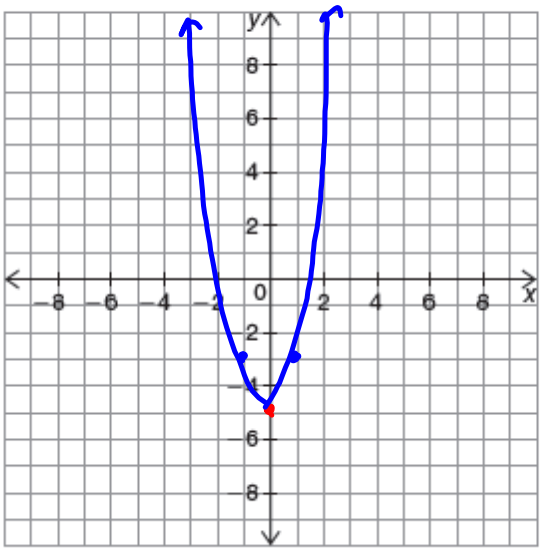
An **odd function** has a graph symmetric about the origin, thus $f(x) = -f(-x)$.

Take 5 mins to work on the problems on pages 342-344 in your book

NOT IN YOUR BOOK

1. Consider the function $f(x) = x^4 + x^2 - 5$.

   a. Graph the function. Verify that the function is even, odd, or neither by comparing 3 pairs of symmetric points and by describing the end behavior of the graph.



end behavior:

as $x \to \infty$, $f(x) \to \infty$

as $x \to -\infty$, $f(x) \to \infty$

   b. Verify algebraically that the function is even, odd, or neither.

even: $f(x) = f(-x)$

$f(x) = x^4 + x^2 - 5$

$\rightarrow f(-x) = (-x)^4 + (-x)^2 - 5$

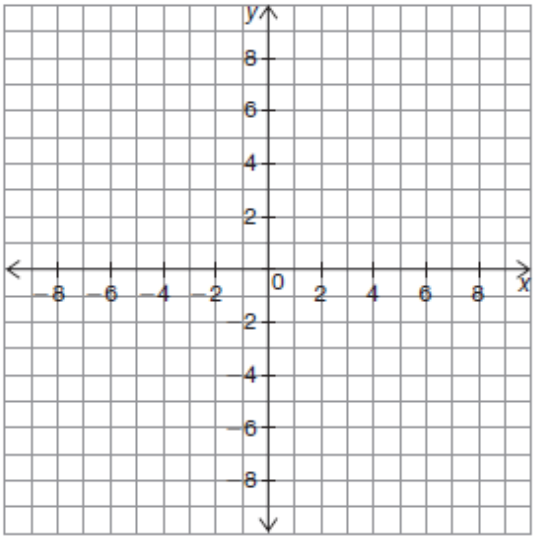
$f(-x) = x^4 + x^2 - 5$

$\Rightarrow f(x) = f(-x)$

NOT IN YOUR BOOK

2. Consider the function $f(x) = x^3 + x$.

   a. Graph the function. Verify that the function is even, odd, or neither by comparing 3 pairs of symmetric points and by describing the end behavior of the graph.



   b. Verify algebraically that the function is even, odd, or neither.

ODD: $f(x) = -f(-x)$

$f(x) = x^3 + x$

$f(-x) = -f(x)$

$f(-x) = (-x)^3 + (-x)$

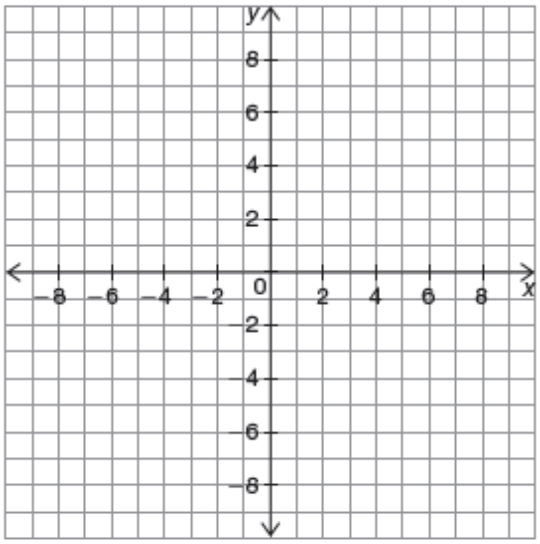
$f(-x) = -x^3 + -x$

$-(x^3 + x)$

NOT IN YOUR BOOK

3. Consider the function $f(x) = x^3 + x^2 - 6x$.

a. Graph the function. Verify that the function is even, odd, or neither by comparing 3 pairs of symmetric points and by describing the end behavior of the graph.



b. Verify algebraically that the function is even, odd, or neither.

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Homework

Finish lesson 5.2

Aug 24-2:00 PM