Questions on 5.1 HW? If not, start working on the problem below...

Looking back . . .

Find all relative extrema for $y = 3x^4 - 16x^3 + 24x^2 + 48$. Be sure to designate which are maxima and which are minima.

$\boxed{5.1 \text{ HW}}$

$$y = \frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)}$$

$x \neq -1, 1$
so -1 & 1
are not
in domain

$y(0) = \frac{1}{0^2-1} = -1$

$(0, -1)$

$$y' = \frac{(x^2-1)(0) - 1(2x)}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$$

$$0 = \frac{-2x}{(x^2-1)^2}$$

$0 = -2x$

$\underline{0 = x}$

critical point:
$x = 0$

* Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

no endpoints, so extreme values occur at critical points. As x moves away from 0 on each side, the y values decrease. So we have a local max at $(0, -1)$.

helpful table:

Critical point	derivative	Type of extrema	value

5.2 Mean Value Theorem (MVT)

Mean Value Theorem (MVT)

If $y = f(x)$ is continuous at every point of the closed interval $[a,b]$ and differentiable at every point of its interior (a,b), then there is at least one point c in (a,b) at which $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.

Example

Show that the function $f(x) = x^2$ satisfies the hypothesis of the Mean Value Theorem on the interval $[\overset{a}{0},\overset{b}{2}]$. Then find a solution c to the equation $f'(c) = \dfrac{\overset{f(2)}{f(b)} - \overset{f(0)}{f(a)}}{b-a}$ on this interval.

$x = c$

$f'(x) = 2x$

$f'(c) = 2c$

$f(0) = 0^2$

$f(0) = 0$

$f(2) = 2^2 = 4$

$2c = \dfrac{4-0}{2-0}$

$2c = \dfrac{4}{2}$

$2c = 2$

$\boxed{c = 1}$

Answer

The function $f(x) = x^2$ is continuous on $[0,2]$ and differentiable on $(0,2)$.

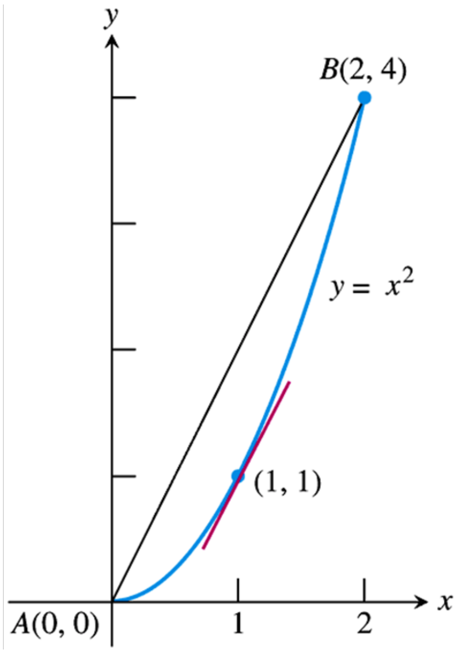
Since $f(0) = 0,\ f(2) = 4,$ and $f'(x) = 2x$

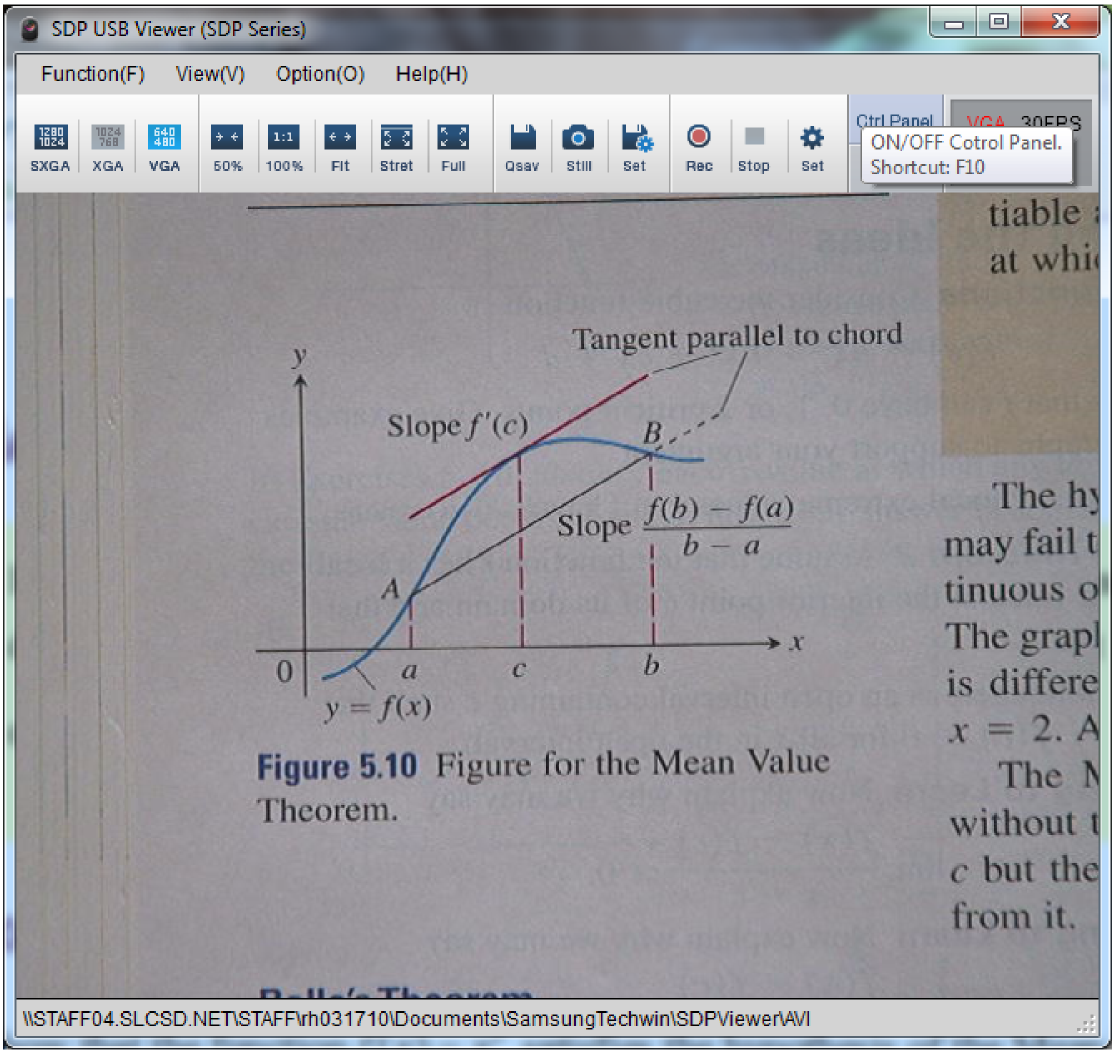
$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$2c = \frac{4-0}{2-0}$$

$$2c = 2$$

$$c = 1.$$

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SXGA XGA VGA 50% 100% Fit Stret Full Qsav Still Set Rec Stop Set
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Shortcut: F10
Tangent parallel to chord
Slope $f'(c)$
B
Slope $\frac{f(b)-f(a)}{b-a}$
A
y
x
0
a
c
b
$y = f(x)$
Figure 5.10 Figure for the Mean Value Theorem.
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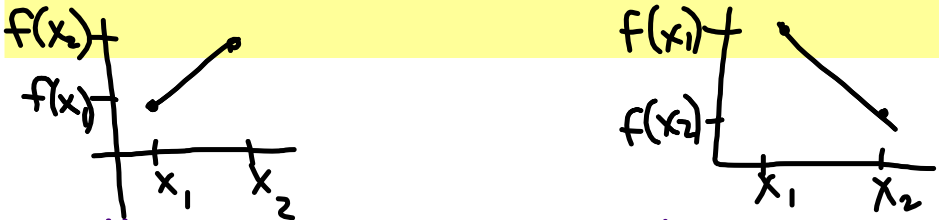
Increasing Function, Decreasing Function

Let f be a function defined on an interval I and let x_1 and x_2 be any two points in I.

1. f **increases** on I if $x_1 < x_2 \quad \Rightarrow \quad f(x_1) < f(x_2)$.
2. f **decreases** on I if $x_1 < x_2 \quad \Rightarrow \quad f(x_1) > f(x_2)$.



Corollary: Increasing and Decreasing Functions

Let f be continuous on $[a,b]$ and differentiable on (a,b).

1. If $f' > 0$ at each point of (a,b), then f increases on $[a,b]$.
2. If $f' < 0$ at each point of (a,b), then f decreases on $[a,b]$.

Example

Where is the function $f(x) = 2x^3 - 12x$ increasing and where is it decreasing?

$f'(x) = 6x^2 - 12$

$6x^2 - 12 > 0$

$\frac{6x^2}{6} > \frac{12}{6}$

$x^2 > 2$

$x > \pm\sqrt{2}$

$x > \sqrt{2}, x < -\sqrt{2}$

increasing

$x > \sqrt{2}$
or
$x < -\sqrt{2}$

$6x^2 - 12 < 0$

$6x^2 < 12$

$x^2 < 2$

$x > -\sqrt{2}$

$x < \sqrt{2}$

decreasing

$-\sqrt{2} < x < \sqrt{2}$

Answer

The function is increasing when $f'(x) > 0$.

$6x^2 - 12 > 0$

$x^2 > 2$

$x < -\sqrt{2}$ or $x > \sqrt{2}$

The function is decreasing when $f'(x) < 0$.

$6x^2 - 12 < 0$

$x^2 < 2$

$-\sqrt{2} < x < \sqrt{2}$

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Corollary: Functions with f' = 0 are constant

If $f'(x)=0$ at each point of an interval I, then there is a constant C for which $f(x)=C$ for all x in I.

Corollary: Functions with the same derivative differ by a constant

If $f'(x)=g'(x)$ at each point of an interval I, then there is a constant C such that $f(x)=g(x)+C$ for all x in I.

Antiderivative

A function $F(x)$ is an **antiderivative** of a function $f(x)$ if $F'(x)=f(x)$ for all x in the domain of f. The process of finding an antiderivative is **antidifferentiation**.

Example

Find the velocity and position functions of a freely falling body for the following set of conditions:

The acceleration is 9.8 m/sec^2 and the body falls from rest.

Answer

Assume that the body is released at time $t = 0$.

Velocity: We know that $a(t) = 9.8$ and $v(0) = 0$, so

$v(t) = 9.8t + C$

$v(0) = 0 + C$

$C = 0$. The velocity function is $v(t) = 9.8t$.

Position: We know that $v(t) = 9.8t$ and $s(0) = 0$, so

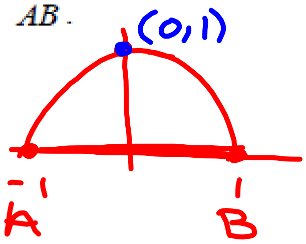
$s(t) = 4.9t^2 + C$

$s(0) = 0 + C$

$C = 0$. The position function is $s(t) = 4.9t^2$.

Examples

2. $f(x)=\sqrt{1-x^2}$, $A=(-1,\ f(-1))$, $B=(1,\ f(1))$. Find a tangent in $(-1,\ 1)$ that is parallel to $\overline{AB}$.



$$f(1)=\sqrt{1-1^2}=\sqrt{1-1}=\sqrt{0}=0$$

$$f(1)=0$$

$$B(1,0)$$

$$f(-1)=\sqrt{1-(-1)^2}=\sqrt{1-1}=\sqrt{0}=0$$

$$f(-1)=0$$

$$A(-1,0)$$

$$m=\frac{0-0}{-1-1}$$

$$m=\frac{0}{-2}$$

$$\boxed{m=0}$$

$$f'(x)=\frac{1}{\cancel{2}}(1-x^2)^{-1/2}(-\cancel{2}x)$$

$$f'(x)=\frac{-x}{\sqrt{1-x^2}}$$

$$0=\frac{-x}{\sqrt{1-x^2}}$$

$$0=-x$$

$$0=x$$

$$f(0)=\sqrt{1-0^2}$$

$$f(0)=\sqrt{1}$$

$$f(0)=1$$

$$(0,1)$$

TANGENT:

$$y-1=0(x-0)$$

$$y=1$$

Homework

5.1 pg.198 #3-30 (X3), 35, 37, 40