Questions on what we've covered so far of Lesson 5.1?

Grab a graphing calculator (we may need to share) and graph the function we were looking at last time: $V(x)=x(8-2x)(10-2x)$ or $V(x)=80x-36x^2+4x^3$

**Make your window go from -10 to 10 for your x-values and -100 to 100 for your y-values.

Feb 27-8:47 AM

5.1 Planting the Seeds
Exploring Cubic Functions

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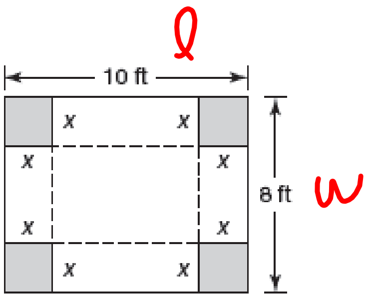
1. Cynthia is an engineer at a manufacturing plant. Her boss asks her to use rectangular metal sheets to build storage bins with the greatest possible volume. Each rectangular sheet is 8 feet by 10 feet.

   Cynthia's sketch shows the squares to be removed from the corners of each sheet. The dashed lines indicate where the metal sheets will be folded before they are welded to form the prism-shaped storage bins without tops.



   a. Complete the table.

Side Length, x, of Removed Squares (ft)	Height of Bin (ft)	Width of Bin (ft)	Length of Bin (ft)	Volume of Bin (ft³)
0	0	8	10	0
1	1	6	8	48
2	2	4	6	48
3	3	2	4	24
4	4	0	2	0
	x	$8-2x$	$10-2x$	

   b. Write a function $V(x)$ to represent the volume of a bin in terms of the side length, x, of the removed squares. Explain your reasoning.

$$V(x) = x(8-2x)(10-2x)$$

Sep 22-8:36 AM

STILL NOT IN YOUR BOOK

c. Represent the function $V(x)$ on a graphing calculator. Determine the domain and range of the function. Determine the domain and range of the function as they relate to the problem situation. Explain your reasoning.

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

In terms of our Volume problem:

D: $[0, 4]$ -OR- D: $(0, 4)$

R: $[0, 52.51]$ R: $(0, 52.51]$

d. Determine the maximum volume of a bin. What are the dimensions of a bin with the maximum volume?

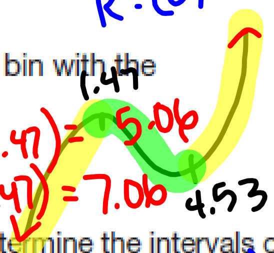
Max. Volume: $52.51 ft^3$

Dimensions: 1.47, 5.06, 7.06

height (x): 1.47

width $(8-2x)$: $8-2(1.47) = 5.06$

length $(10-2x)$: $10-2(1.47) = 7.06$



e. Determine any relative maximums or relative minimums of $V(x)$. Then, determine the intervals over which the function is increasing and decreasing.

relative min: $(4.53, -4.51)$

relative max: $(1.47, 52.51)$

Increasing: $(-\infty, 1.47)$
$(4.53, \infty)$

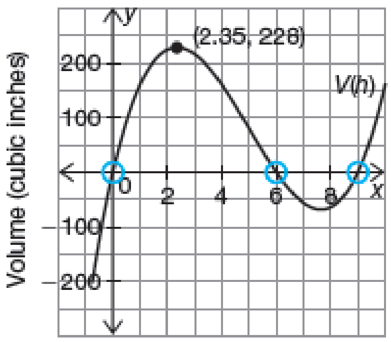
Decreasing: $(1.47, 4.53)$

f. Determine the *x*- and *y*-intercepts of the graph of $V(x)$. What do they represent in this problem situation?

g. Cynthia's boss asks her to make several bins with volumes of exactly 40 cubic feet. Determine the bin dimensions that will work.

PG.320 IN YOUR BOOK

The key characteristics of a function may be different within a given domain. The function $V(h) = h(12 - 2h)(18 - 2h)$ has x-intercepts at $x = 0$, $x = 6$, and $x = 9$.



As the input values for height increase, the output values for volume approach infinity. Therefore, the function doesn't have a maximum; however, the point (2.35, 228) is a *relative maximum* within the domain interval of (0, 6). A **relative maximum** is the highest point in a particular section of a graph. Similarly, as the values for height decrease, the output values approach negative infinity. Therefore, a *relative minimum* occurs at (7.65, −68.16). A **relative minimum** is the lowest point in a particular section of a graph.

The function $v(h)$ represents all of the possible volumes for a given height h. A horizontal line is a powerful tool for working backwards to determine the possible values for height when the volume is known.

Oct 9-8:14 AM

PG.326 IN YOUR BOOK

The volume functions for the rectangular planter box and the cylindrical shaped planter are examples of *cubic functions*. A **cubic function** is a function that can be written in the standard form $f(x) = ax^3 + bx^2 + cx + d$ where $a \neq 0$. In other words, a cubic function is a polynomial function of degree 3.

The volume of the rectangular planter box was represented as $V(h) = h(12 - 2h)(18 - 2h)$. You can multiply the three factors to express the function in standard cubic form.

$$V(h) = h(12 - 2h)(18 - 2h)$$
$$= h(216 - 60h + 4h^2)$$
$$= 216h - 60h^2 + 4h^3$$
$$V(h) = 4h^3 - 60h^2 + 216h$$

The volume of the cylindrical shaped planter was represented as $V(x) = (3.14x^2)(2x)$. You can multiply the two factors to express the function in standard cubic form.

$$V(x) = 6.28x^3$$

The Fundamental Theorem of Algebra tells you that a cubic function must have 3 zeros. Roots may be any number in the set of complex numbers, and can even appear multiple times. **Multiplicity** is how many times a particular number is a zero for a given polynomial function. For example in the polynomial function that represents the volume of the cylindrical shaped planter, $V(x) = (3.14x^2)(2x)$, the zero, $x = 0$, has multiplicity 3.

$f(x) = x^2(x+1)$ $f(x) = (x-1)^3$

$f(x) = (x-2)^2(x-1)$

$f(x) = (x-2)(x-2)(x-1)$

Oct 21-8:47 AM

PG.329 IN YOUR BOOK

3. Determine each product. Show all your work and then use a graphing calculator to verify your product is correct.

   a. $(x + 2)(-3x + 2)(1 + 2x)$

Oct 21-8:49 AM

NOT IN YOUR BOOK

2. Write a cubic function with zeros of -4, 2, and 3. Write the function in the form $f(x) = ax^3 + bx^2 + cx + d$. Verify graphically that the function has the correct zeros.

3. Consider the given functions.

- $f(x) = x + 2$
- $g(x) = x^2 - 3.5x + 2.5$
- $h(x) = f(x) \cdot g(x) = (x + 2)(x^2 - 3.5x + 2.5)$

a. Determine the zeros of $f(x)$, $g(x)$, and $h(x)$.

b. How are the zeros of $h(x)$ related to the zeros of $f(x)$ and $g(x)$. Explain why this is true.

c. Write a function $m(x)$ that has the same zeros as $h(x)$ plus an additional zero of 5. Verify your answer graphically.

Homework

Finish lesson 5.1