Questions on 4.3 HW?

15, 27, 12

(12) $x(t) = \tan^{-1}(t^2)$

$v(t) = \frac{1}{1+t^4} \cdot 2t = \frac{2t}{1+t^4}$

$v(1) = \frac{2 \cdot 1}{1+1^4} = \frac{2}{2} = 1$

(15) $y = \csc^{-1}(x^2+1), x>0$

$y = \frac{\pi}{2} - \sec^{-1}(x^2+1), x>0$

$y' = -\frac{1}{|x^2+1|\sqrt{(x^2+1)^2-1}} \cdot 2x$

$y' = \frac{-2x}{|x^2+1|\sqrt{x^4+2x^2+\cancel{1}-\cancel{1}}} = \frac{-2\cancel{x}}{|x^2+1| \cdot \cancel{x} \cdot \sqrt{x^2+2}}$

$\sqrt{x^2(x^2+2)} = \sqrt{x^2}\sqrt{x^2+2}$

$y' = \frac{-2}{|x^2+1|\sqrt{x^2+2}}$

(27) $y = \tan x$ @ $\left(\frac{\pi}{4}, 1\right)$ $x>0$

a) $y' = \sec^2 x$

$y'\left(\frac{\pi}{4}\right) = \sec^2\left(\frac{\pi}{4}\right) = \frac{1}{\cos^2\left(\frac{\pi}{4}\right)} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{4}{2} = \boxed{2}$

$y - 1 = 2\left(x - \frac{\pi}{4}\right)$

$\boxed{y = 2x - \frac{\pi}{2} + 1}$

(b) $y = \tan^{-1}(x)$

$y' = \frac{1}{1+x^2} \rightarrow y'(1) = \frac{1}{2}$

$y - \frac{\pi}{4} = \frac{1}{2}(x-1)$

$\boxed{y = \frac{1}{2}x - \frac{1}{2} + \frac{\pi}{4}}$

4.4 Derivatives of Exponential and Logarithmic Functions

Derivative of e^x

If u is a differentiable function of x, then

$$\frac{d}{dx}e^u = e^u \frac{du}{dx}$$

Examples

1. Find y' if $y = e^{-x}$

$y' = e^{-x} \cdot -1 = \boxed{-e^{-x}}$

2. Find y' if $y = e^{2x-1}$.

$y' = e^{2x-1} \cdot 2 = \boxed{2e^{2x-1}}$

3. Find $\frac{d}{dx}$ if $f(x) = e^{\frac{3}{x}}$. $= e^{3x^{-1}}$

$\frac{d}{dx} = e^{\frac{3}{x}} \cdot -3x^{-2}$

$\frac{d}{dx} = \boxed{-\frac{3e^{\frac{3}{x}}}{x^2}}$

4. $\frac{d}{dx}\left(y = x^2 e^{-x}\right) =$

$\frac{dy}{dx} = x^2 \cdot -e^{-x} + e^{-x} \cdot 2x$

$\frac{dy}{dx} = \boxed{-x^2e^{-x} + 2xe^{-x}}$

Derivative of a^x

u is a differentiable function of x and for $a > 0$ and $a \neq 1$,

$$\frac{d}{dx}\left(a^u\right) = a^u \ln a \frac{du}{dx}$$

Examples

6. Find the derivatives of the following functions:

a. $y = 2^{3x}$

$y' = 2^{3x} \cdot \ln 2 \cdot 3$

$\boxed{y' = 3 \cdot 2^{3x} \cdot \ln 2}$

b. $y = 3^{\cot x}$

$y' = 3^{\cot x} \cdot \ln 3 \cdot (-\csc^2 x)$

$\boxed{y' = -\ln 3 (\csc^2 x) 3^{\cot x}}$

~~c. $g(t) = t^2 2^t$~~

c) $g(t) = t^2 \cdot 2^t$

$g'(t) = t^2 \cdot 2^t \cdot \ln 2 \cdot 1 + 2^t \cdot 2t$

$g'(t) = \boxed{t^2 \cdot 2^t \cdot \ln^2 + 2^{t+1} \cdot t}$

7. At what point on the graph of the function $y = 2^t - 3$ does the tangent line have a slope of 21?

$y' = 2^t \cdot \ln 2 - 0$

$7 = 2^t$

$$\frac{21}{\ln 2} = \frac{2^t \cdot \ln 2}{\ln 2}$$

$$\frac{21}{\ln 2} = 2^t$$

$\ln (30.3) = \ln 2^t$

$$\frac{\ln(30.3)}{\ln 2} = \frac{t \cancel{\ln 2}}{\cancel{\ln 2}}$$

$4.9 = t$

$y(4.9) = 2^{4.9} - 3$

$y(4.9) = 26.9$

$(4.9, 26.9)$

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Derivative of ln x (natural logarithm)

If u is a differentiable function of x and $u > 0$,

$$\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}$$

Examples

Find the derivatives of the following functions:

$f(x) = \ln(x^2)$

$f'(x) = \frac{1}{x^2} \cdot 2x$

$= \frac{2x}{x^2} = \boxed{\frac{2}{x}}$

b. $y = \ln(2 - \cos x)$

$y' = \frac{1}{2-\cos x} \cdot \sin x$

$y' = \boxed{\frac{\sin x}{2-\cos x}}$

c. $y = x\ln x - x$

$y' = x \cdot \frac{1}{x} + \ln x \cdot 1 - 1$

$= 1 + \ln x - 1$

$= \boxed{\ln x}$

Derivative of $\log_a x$

If u is a differentiable function of x and $u > 0$,

$$\frac{d}{dx}\log_a u = \frac{1}{u\ln a}\frac{du}{dx}$$

Examples

9. Find the derivatives of the following functions:

a. $x(t) = \log_{\underset{a}{2}}(\underset{u}{3t+1})$

$$x'(t) = \frac{1}{(3t+1)\ln 2}\cdot 3$$

$$= \boxed{\frac{3}{(3t+1)\ln 2}}$$

b. $y = \dfrac{1}{\log_5 x} = (\log_5 x)^{-1}$

$$y' = -1(\log_5 x)^{-2}\left(\frac{1}{x\ln 5}\cdot 1\right)$$

$$= \boxed{-\frac{1}{(\log_5 x)^2 x\ln 5}}$$

c. $f(x) = \log e^x$

$$f(x) = \log_{10} e^x$$

$$f'(x) = \frac{1}{e^x\ln 10}\cdot e^x$$

$$f'(x) = \frac{\cancel{e^x}}{\cancel{e^x}\ln 10}$$

$$= \boxed{\frac{1}{\ln 10}}$$

10. Find $\dfrac{dy}{dx}$ if $y = \log_{\underset{a}{7}}(\underset{u}{7^{\sin x}})$.

$$\frac{dy}{dx} = \frac{1}{7^{\sin x}(\ln 7)}\cdot \underbrace{7^{\sin x}\cdot\ln 7\cdot\cos x}_{\frac{du}{dx}}$$

$$\frac{dy}{dx} = \frac{\cancel{7^{\sin x}}\cdot\cancel{\ln 7}\cdot\cos x}{\cancel{7^{\sin x}}\cdot\cancel{\ln 7}}$$

$$= \boxed{\cos x}$$

Logarithmic Differentiation

Sometimes the properties of logarithms can be used to simplify the differentiation process, even if logarithms themselves must be introduced as a step in the process.

The process of introducing logarithms before differentiating is called *logarithmic differentiation.*

Homework

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