Questions on 4.2 HW?

24) $x \sin 2y = y \cos 2x \rightarrow (\pi/4, \pi/2)$

$$x \cdot \cos 2y \cdot 2\frac{dy}{dx} + \sin 2y \cdot 1 = y \cdot -\sin 2x \cdot 2 + \cos 2x \cdot \frac{dy}{dx}$$

$$2x\cos 2y\frac{dy}{dx} + \sin 2y = -2y\sin 2x + \cos 2x\frac{dy}{dx}$$

$$\sin 2y + 2y\sin 2x = \cos 2x\frac{dy}{dx} - 2x\cos 2y\frac{dy}{dx}$$

$$\frac{\sin 2y + 2y\sin 2x}{\cos 2x - 2x\cos 2y} = \frac{\cancel{(\cos 2x - 2x\cos 2y)}\frac{dy}{dx}}{\cancel{\cos 2x - 2x\cos 2y}}$$

$$\boxed{\frac{\sin 2y + 2y\sin 2x}{\cos 2x - 2x\cos 2y} = \frac{dy}{dx}}$$

@ $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$$\frac{\sin 2\left(\frac{\pi}{2}\right) + 2\left(\frac{\pi}{2}\right)\sin 2\left(\frac{\pi}{4}\right)}{\cos 2\left(\frac{\pi}{4}\right) - 2\left(\frac{\pi}{4}\right)\cos\left(2\cdot\frac{\pi}{2}\right)} = \frac{\sin\pi + \pi\sin\left(\frac{\pi}{2}\right)}{\cos\left(\frac{\pi}{2}\right) - \frac{\pi}{2}\cos\pi}$$

$$= \frac{0 + \pi\cdot 1}{0 - \frac{\pi}{2}\cdot -1} = \frac{2\pi}{\pi} = \boxed{2}$$

a) tangent:

$$y - \frac{\pi}{2} = 2\left(x - \frac{\pi}{4}\right)$$

$$y = 2x - \frac{\pi}{2} + \frac{\pi}{2}$$

$$\boxed{y = 2x}$$

b) normal:

$$y - \frac{\pi}{2} = -\frac{1}{2}\left(x - \frac{\pi}{4}\right)$$

$$y = -\frac{1}{2}x + \frac{\pi}{8} + \frac{\pi}{2}$$

$$\boxed{y = -\frac{1}{2}x + \frac{5\pi}{8}}$$

y^3

$3y^2 \cdot \frac{dy}{dx}$

42) $y = [\sin(x+5)]^{5/4}$

$$y' = \frac{5}{4}(\sin(x+5)]^{1/4} \cdot (\cos(x+5)) \cdot 1$$

$$y' = \frac{5}{4}[\sin(x+5)]^{1/4} \cdot (\cos(x+5))$$

or

$$y' = \frac{5}{4}\sqrt[4]{\sin(x+5)} \cdot \cos(x+5)$$

4.3 Derivatives of Inverse Trigonometric Functions

Derivatives of Inverse Functions

If f is differentiable at every point of an interval I and $\frac{dy}{dx}$ is never zero on I, then f has an inverse and f^{-1} is differentiable at every point on the interval $f(I)$.

Derivative of the Arcsine

If u is a differentiable function of x with $|u|<1$, we apply the Chain Rule to get

$$\frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \quad |u|<1.$$

arcsin(   )

Derivative of the Arctangent

The derivative is defined for all real numbers.

If u is a differentiable function of x, we apply the Chain Rule to get

$$\frac{d}{dx}\tan^{-1}u=\frac{1}{1+u^2}\frac{du}{dx}.$$

Derivative of the Arcsecant

If u is a differentiable function of x with $|u|>1$, we have the formula

$$\frac{d}{dx}\sec^{-1}u=\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx},\quad |u|>1.$$

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Inverse Cofunction Identities

$$\cos^{-1}x=\frac{\pi}{2}-\sin^{-1}x$$

$$\cot^{-1}x=\frac{\pi}{2}-\tan^{-1}x$$

$$\csc^{-1}x=\frac{\pi}{2}-\sec^{-1}x$$

Find y'

$y=\cos^{-1}(x)$

$y=\frac{\pi}{2}-\sin^{-1}x$

$y'=0-\frac{1}{\sqrt{1-x^2}}\cdot 1$

$$\sec^{-1}x=\cos^{-1}\left(\frac{1}{x}\right)$$

$$\cot^{-1}x=\frac{\pi}{2}-\tan^{-1}x$$

$$\csc^{-1}x=\sin^{-1}\left(\frac{1}{x}\right)$$

Calculator Conversion Identities

$\frac{d}{dx}\cos^{-1}x=-\frac{d}{dx}\sin^{-1}x$

Examples

1. $\frac{d}{dx}\left(\sin^{-1} x^2\right) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$

$x^2 \uparrow u$, $(x^2)^2 \uparrow u^2$

2. If $x(t) = \sin^{-1}(1-t)$, then $x'(t) = \frac{1}{\sqrt{1-(1-t)^2}} \cdot -1 = -\frac{1}{\sqrt{1-(1-2t+t^2)}}$

$(1-t)^2 \to 1-2t+t^2$

$= -\frac{1}{\sqrt{2t-t^2}}$

3. Find $\frac{dy}{dx}$ if $y = \cos^{-1}\left(\frac{1}{x}\right)$. $\to$ use: $\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$

$y = \sec^{-1} x$

so... $y' = \frac{1}{|x|\sqrt{x^2-1}} \cdot 1 = \frac{1}{|x|\sqrt{x^2-1}}$

4. A particle moves along the x-axis so that its position at any time $t \geq 0$ is given by the function $x = \tan^{-1}\sqrt{t}$. What is the particle's velocity at $t = 16$?

$v(t) = \frac{1}{1+(t^{1/2})^2} \cdot \frac{1}{2\sqrt{t}} = \frac{1}{(2\sqrt{t})(1+t)}$

$v(16) = \frac{1}{8 \cdot 17} = \frac{1}{136}$

5. Find an equation for the line tangent to the graph of $f(x) = \tan^{-1} x$ at $x = 1$.

$f'(x) = \frac{1}{1+x^2}$ $\qquad f'(1) = \frac{1}{2}$ $\qquad f(1) = \tan^{-1}(1) = \frac{\pi}{4}$

$\left(1, \frac{\pi}{4}\right)$

$y - \frac{\pi}{4} = \frac{1}{2}(x-1)$

$\boxed{y = \frac{1}{2}x - \frac{1}{2} + \frac{\pi}{4}}$

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Homework

4.3 pg.175-6 #3-27(X3); 37-40all