Questions on the Chain Rule?

4.2 Implicit Differentiation

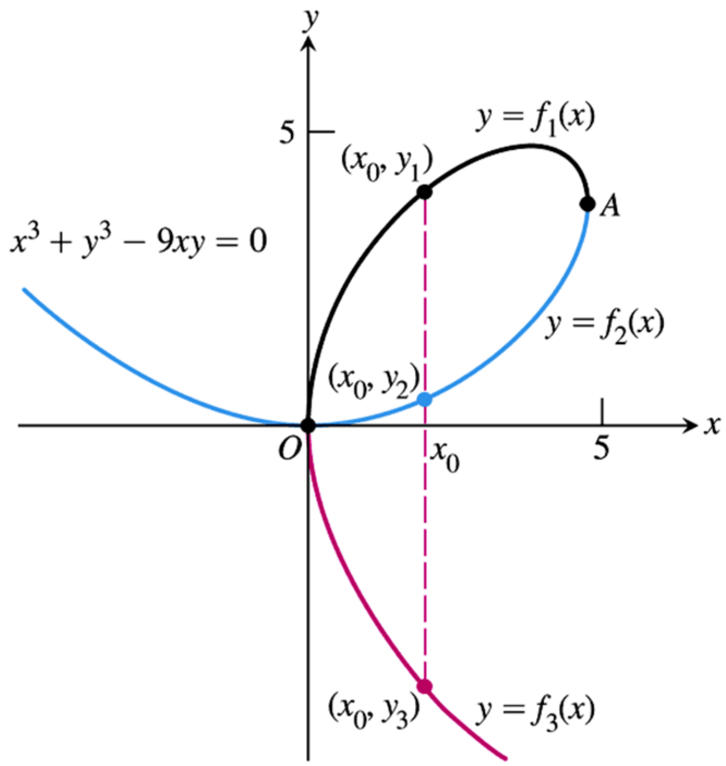
An important problem in Calculus is how to find the slope when the function can't conveniently be solved for y. Instead, y is treated as a differentiable function of x and both sides of the equation are differentiated with respect to x, using the appropriate rules for sums, products, quotients and the Chain rule. Then solve for $\frac{dy}{dx}$ in terms of x and y together to obtain a formula that calculates the slope at any point (x, y) on the graph.

The process by which we find $\frac{dy}{dx}$ is called **implicit differentiation**.

The phrase derives from the fact that the equation

$$x^3 + y^3 - 9xy = 0$$

defines the functions f_1, f_2 and f_3 implicitly (i.e., hidden inside the equation), without giving us *explicit* formulas to work with.



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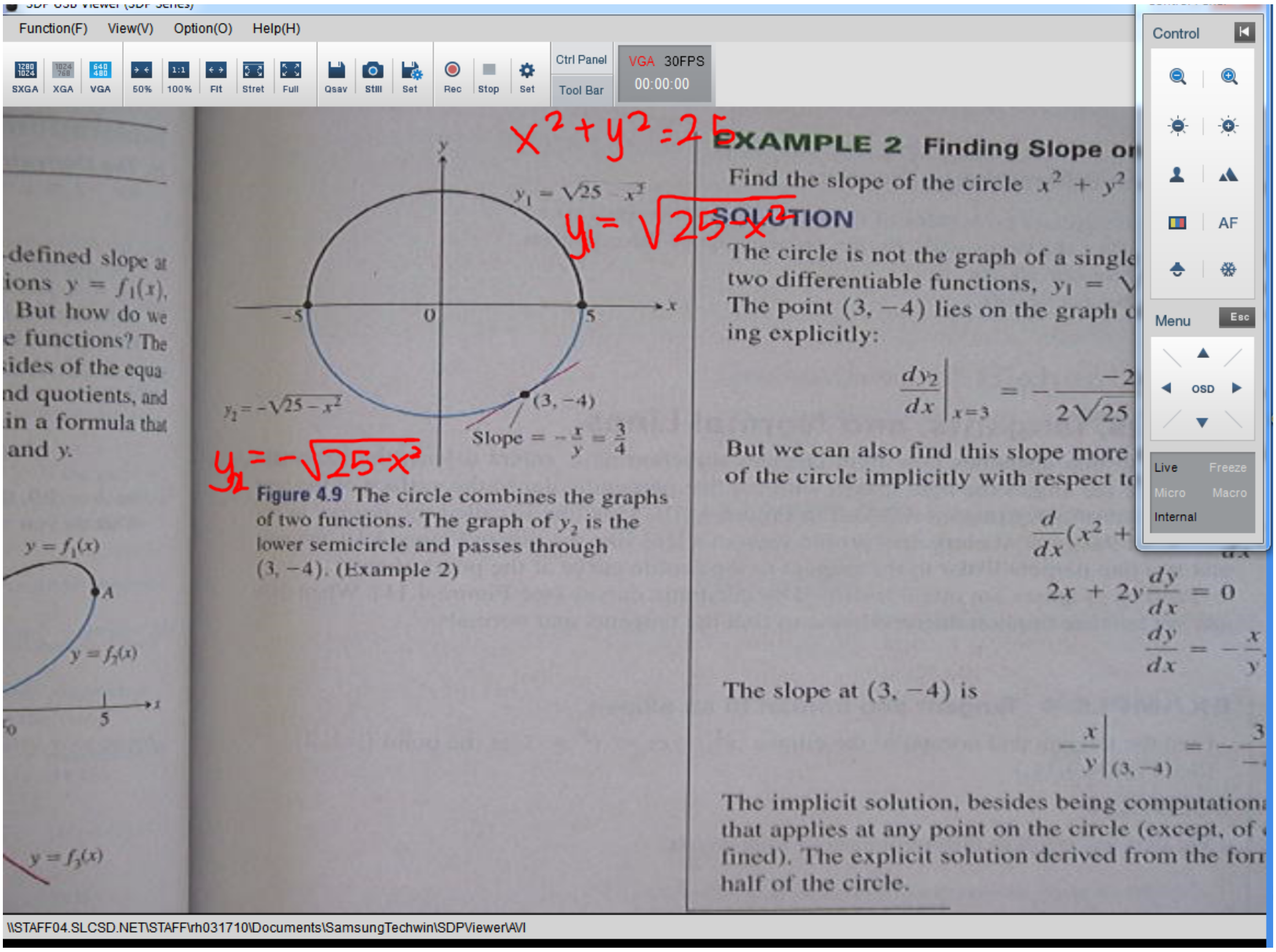

$x^2+y^2=25$

EXAMPLE 2 Finding Slope on

Find the slope of the circle $x^2 + y^2$

$y_1 = \sqrt{25 - x^2}$

$y_1 = \sqrt{25-x^2}$

SOLUTION

The circle is not the graph of a single
two differentiable functions, $y_1 = \sqrt{}$
The point $(3, -4)$ lies on the graph o
ing explicitly:

$$\left.\frac{dy_2}{dx}\right|_{x=3} = -\frac{-2}{2\sqrt{25}}$$

$y_2 = -\sqrt{25 - x^2}$

$(3, -4)$

Slope $= -\frac{x}{y} = \frac{3}{4}$

$y_2 = -\sqrt{25-x^2}$

Figure 4.9 The circle combines the graphs of two functions. The graph of y_2 is the lower semicircle and passes through $(3, -4)$. (Example 2)

But we can also find this slope more
of the circle implicitly with respect to

$$\frac{d}{dx}(x^2 +$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

The slope at $(3, -4)$ is

$$\left.-\frac{x}{y}\right|_{(3,-4)} = -\frac{3}{-4}$$

The implicit solution, besides being computation
that applies at any point on the circle (except, of
fined). The explicit solution derived from the for
half of the circle.

defined slope at
ions $y = f_1(x)$.
But how do we
e functions? The
sides of the equa
nd quotients, and
in a formula that
and y.

$y = f_1(x)$

$y = f_2(x)$

$y = f_3(x)$

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Implicit Differentiation Process

1. Differentiate both sides of the equation with respect to x.
2. Collect the terms with $\frac{dy}{dx}$ on one side of the equation.
3. Factor out $\frac{dy}{dx}$.
4. Solve for $\frac{dy}{dx}$.

$\frac{dy}{dx}$ or y'

Examples

1. $\frac{d}{dx}\left(x^2 - 2xy + y^2 = 4\right)$

$(x-y)^2=4$

$$\frac{d}{dx}x^2 - \frac{d}{dx}2xy + \frac{d}{dx}y^2 = \frac{d}{dx}4$$

$$2x - 2\left(x\frac{dy}{dx} + y\cdot 1\right) + 2y\frac{dy}{dx} = 0$$

$$2x - 2x\frac{dy}{dx} - 2y + 2y\frac{dy}{dx} = 0$$

$$2x - 2y = 2x\frac{dy}{dx} - 2y\frac{dy}{dx}$$

$$\frac{2x-2y}{2x-2y} = \frac{\cancel{(2x-2y)}\frac{dy}{dx}}{\cancel{2x-2y}}$$

$$1 = \frac{2x-2y}{2x-2y} = \frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = 1}$$

2. Find $\frac{dy}{dx}$ for $2y = x^2 + \sin y$.

$$2\cdot 1\cdot\frac{dy}{dx} = 2x + \cancel{\cos y\frac{dy}{dx}}$$

$$-\cos y\frac{dy}{dx} \qquad \cancel{-\cos y\frac{dy}{dx}}$$

$$2\frac{dy}{dx} - \cos y\frac{dy}{dx} = 2x$$

$$\frac{(2-\cos y)\frac{dy}{dx}}{2-\cos y} = \frac{2x}{2-\cos y}$$

$$\boxed{\frac{dy}{dx} = \frac{2x}{2-\cos y}}$$

3. Find $\frac{dy}{dx}$ for the function $x^2 + xy + y^2 = 7$.

$$\frac{dy}{dx} = \frac{2x+y}{-2y-x}$$

$$2x + xy' + y\cdot 1 + 2yy' = 0$$

$$2x + y = -2yy' - xy'$$

$$\frac{2x+y}{-2y-x} = \frac{(-2y-\cancel{x})y'}{-2\cancel{y}-x}$$

$$\boxed{\frac{2x+y}{-2y-x} = y'}$$

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AP Problems!!

4. If $\sin(xy) = x$ then $\frac{dy}{dx} =$

(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x\cos(xy)}$

(C) $\frac{1-\cos(xy)}{\cos(xy)}$

(D) $\frac{1-y\cos(xy)}{x\cos(xy)}$

(E) $\frac{y(1-\cos(xy))}{x}$

$\cos(xy)\cdot(xy'+y\cdot 1)=1$

$\cos(xy)\cdot xy' + \cos(xy)\cdot y = 1$

$\frac{\cancel{x}y'\cdot\cancel{\cos(xy)}}{\cancel{x\cos(xy)}} = \frac{1-y\cos(xy)}{x\cos(xy)}$

$y' = \frac{1-y\cos(xy)}{x\cos(xy)}$

5. If $4x^2 + 2xy + 3y = 9$, then the value of $\frac{dy}{dx}$ at the point $(2, -1)$ is

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 2 (D) -2 (E) none of these

$8x + 2(xy' + y) + 3y' = 0$

$8x + 2xy' + 2y + 3y' = 0$

$\frac{8x+2y}{-2x-3} = \frac{(-2x-3)y'}{-2x-3}$

$\boxed{\frac{8x+2y}{-2x-3} = y'}$

$(2,-1) \rightarrow \frac{8\cdot 2 + 2\cdot -1}{-2\cdot 2 - 3} = \frac{14}{-7} = \boxed{-2}$

Slopes and tangent lines:

Write the equation(s) of the tangent(s) to $x^2 + xy - y^2 = 1$ at the point where $x = 2$.

$2x + xy' + y - 2yy' = 0$

$\frac{-2x + y}{2y - x} = \frac{(2y-x)y'}{2y-x}$

$\boxed{\frac{2x+y}{2y-x} = y'}$

$2^2 + 2y - y^2 = 1$

$4 + 2y - y^2 = 1 \quad -4$

$0 = y^2 - 2y - 3$

$0 = (y-3)(y+1)$

$\boxed{y = 3, -1}$

$(2, 3)$

$(2, -1)$

$(2,3) \rightarrow \frac{2 \cdot 2 + 3}{2 \cdot 3 - 2} = \frac{7}{4}$

$y - 3 = \frac{7}{4}(x - 2)$

$y = \frac{7}{4}x - \frac{7}{2} + 3$

$y = \frac{7}{4}x - \frac{1}{2}$

$(2,-1) \rightarrow \frac{2 \cdot 2 + -1}{2 \cdot -1 - 2} = \frac{3}{-4}$

$y + 1 = -\frac{3}{4}(x - 2)$

$y = -\frac{3}{4}x + \frac{3}{2} - 1$

$y = -\frac{3}{4}x + \frac{1}{2}$

Second derivatives:

Find $\frac{d^2y}{dx^2}$ of $2x^3 - 3y^2 = 8$

$6x^2 - 6yy' = 0$

$\frac{6x^2}{6y} = \frac{6yy'}{6y}$

$\boxed{\frac{x^2}{y} = y'}$

2nd der:

$\frac{y \cdot 2x - x^2 \cdot y'}{y^2} =$

$\frac{2xy - x^2\left(\frac{x^2}{y}\right)}{y^2} = \frac{y}{y} \cdot \frac{2xy - \frac{x^4}{y}}{y^2} = \frac{\frac{2xy^2 - x^4}{y}}{y^2}$

$= \boxed{\frac{2xy^2 - x^4}{y^3}}$

Homework

4.2 pg.167 #3-42 (X3)