Questions on 4.1a HW?

12, 27

(12) $s = \sin\left(\frac{3\pi}{2}t\right) + \cos\left(\frac{7\pi}{4}t\right)$

$v(t) = s' = \cos\left(\frac{3\pi}{2}t\right)\cdot\left(\frac{3\pi}{2}\right) - \sin\left(\frac{7\pi}{4}t\right)\cdot\left(\frac{7\pi}{4}\right)$

$\boxed{v(t) = \frac{3\pi}{2}\cos\left(\frac{3\pi}{2}t\right) - \frac{7\pi}{4}\sin\left(\frac{7\pi}{4}t\right)}$

(27) $r = \sqrt{\theta\cdot\sin\theta} = (\theta\sin\theta)^{1/2}$

$r' = \frac{1}{2}(\theta\sin\theta)^{-1/2}\cdot(\theta\cos\theta + \sin\theta\cdot 1)$

$\boxed{r' = \frac{\theta\cos\theta + \sin\theta}{2\sqrt{(\theta\sin\theta)}}}$

(18) $y = 4\sqrt{\sec x + \tan x} = 4(\sec x + \tan x)^{1/2}$

$y' = 2(\sec x + \tan x)^{-1/2}\cdot(\sec x\tan x + \sec^2 x)$

$= \frac{2(\sec x\tan x + \sec^2 x)}{\sqrt{\sec x + \tan x}}$

$= \frac{2\sec x(\tan x + \sec x)^1}{\sqrt{\sec x + \tan x}} = 2\sec x(\tan x + \sec x)^{1-1/2}$

$= \boxed{2\sec x\sqrt{\tan x + \sec x}}$

4.1b More Chain Rule

3. Write the equation of the line tangent to $f(x)=\sqrt{3x^2-2}$ at the point where $x=3$.

$f(3)=\sqrt{3(3)^2-2}=5$

$(3,5)$

slope: $9/5$

$f'(x)=\sqrt{3x^2-2}=(3x^2-2)^{1/2}$

$=\frac{1}{2}(3x^2-2)^{-1/2}\cdot 6x$

$f'(x)=\frac{6x}{2\sqrt{3x^2-2}}=\boxed{\frac{3x}{\sqrt{3x^2-2}}}$

$f'(3)=\frac{3\cdot 3}{\sqrt{3(3)^2-2}}=\frac{9}{5}$

$y-5=\frac{9}{5}(x-3)$

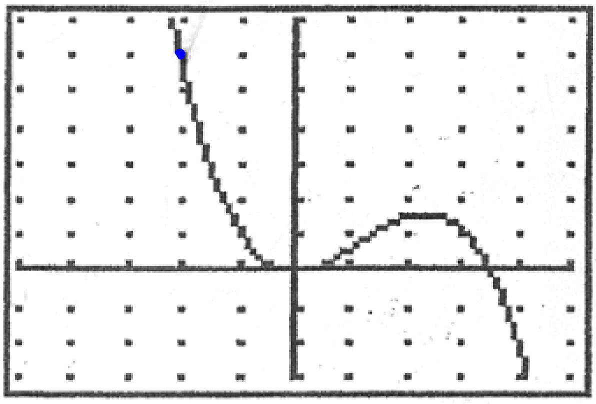
$y=\frac{9}{5}x-\frac{27}{5}+5$

$\boxed{y=\frac{9}{5}x-\frac{2}{5}}$

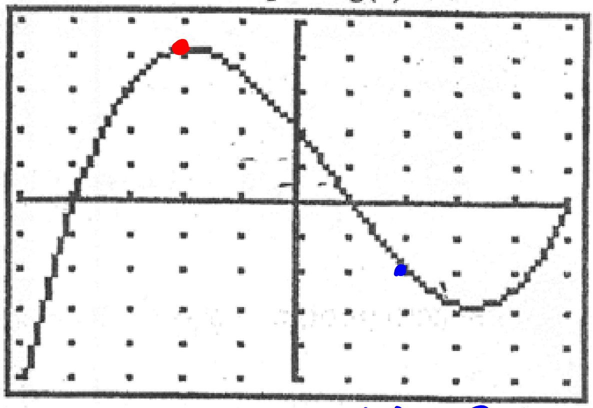
The Chain Rule graphically:

Let f and g be the functions defined below

Graph of f(x)



Graph of g(x)



$g(2)=-2$

$f(-2)=6$

Let h(x) = f(g(x))

a) Evaluate h(-2), h(1), and h(2)

$h(-2)=f(g(-2))$

$h(0)=1.5$ $h(-2)=f(4)$

$h(-2)=-2$

$h(1)=f(g(1))$

$h(1)=f(0)$

$h(1)=0$

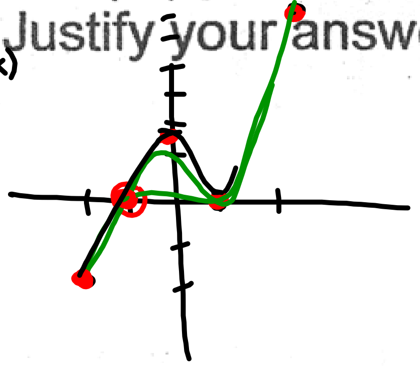
$h(2)=f(g(2))$

$h(2)=f(-2)$

$h(2)=6$

b) Is h'(-1) positive, negative, or equal to zero. Justify your answer.

h(x)



$h(-1)=f(g(-1))$

$=f(3.5)$

$=0$

* Positive; it is increasing

c) Estimate the sign of h'(-2), h'(1), and h'(2).

4.1b WKS

(13) $x = \sin 2\pi t \qquad y = \cos 2\pi t \qquad t = -\frac{1}{6}$

$x = \sin 2\pi\left(-\frac{1}{6}\right) \qquad y = \cos 2\pi\left(-\frac{1}{6}\right)$

$x = \sin\left(\frac{-\pi}{3}\right) \qquad y = \cos\left(\frac{-\pi}{3}\right) \qquad \left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$

$x = -\frac{\sqrt{3}}{2} \qquad y = \frac{1}{2}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$\frac{dx}{dt} = (\cos 2\pi t)\cdot 2\pi \qquad \frac{dy}{dt} = (-\sin 2\pi t)(2\pi)$

$\frac{dx}{dt} = 2\pi \cos 2\pi t \qquad \frac{dy}{dt} = -2\pi \sin 2\pi t$

$$\frac{dy}{dx} = \frac{-\cancel{2\pi} \sin 2\pi t}{\cancel{2\pi} \cos 2\pi t} = -\frac{\sin 2\pi t}{\cos 2\pi t} = \boxed{-\tan 2\pi t}$$

$$\left.\frac{dy}{dx}\right|_{t=-1/6} = -\tan 2\pi\left(-\frac{1}{6}\right)$$

$$= -\tan\left(-\pi/3\right) = -\frac{\sin\left(\frac{-\pi}{3}\right)}{\cos\left(\frac{-\pi}{3}\right)}$$

$$= -\left(\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right) = -\left(-\frac{\sqrt{3}}{2}\cdot\frac{2}{1}\right) = \boxed{\sqrt{3}}$$

☆EQN: $y - \frac{1}{2} = \sqrt{3}\left(x + \frac{\sqrt{3}}{2}\right)$

$y = \sqrt{3}x + \frac{3}{2} + \frac{1}{2}$

$y = \sqrt{3}x + 2$

Practice

Differentiate each function with respect to the given variable.

$f(w)=(2w^2+1)^2$

$r=(-3x^4-5)^5$

$f(x)=(5x^5-1)^{\frac{1}{2}}(2x+5)$

$f(x)=(5x+3)^{-5}$

$f(x)=\left(\sqrt[3]{x^3+5}-2\right)^5$

$y=(-4x^4+5)^{\frac{1}{5}}$

$y=(2x+1)^{\frac{1}{5}}$

$f(x)=\sqrt[5]{-3x^4-1}$

$y=\sin 2x^5$

$y=\cos x^4$

$y=\csc\left(\sin 2x^2\right)$

$y=\tan 5x^4$

$y=\cos\left(\csc 3x^4\right)$

$y=\sec x^2$

$y=\left(\dfrac{-3x^2-2}{4x^3+3}\right)^3$

$y=\dfrac{(-5x-1)^4}{4x^3-5}$

Homework

4.1b WKS