B1: Finish these problems on pages 49-51 in your book - you have 15 minutes - and I will be checking your 3.6 and 3.7 HW soon, before we go over 3.8 HW questions.

3.8 To Be Determined . . .

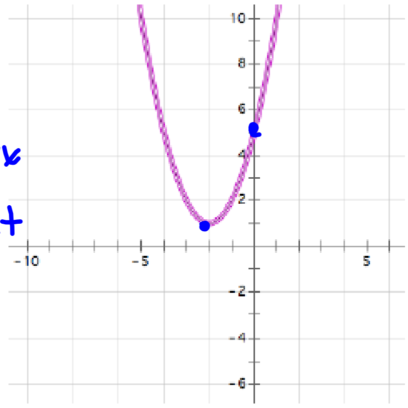
A Develop Understanding Task

4.

x	y
-5	10
-4	5
-3	2
-2	1
-1	2
0	5
1	10
2	17
3	26
4	37
5	50

vertex

y-int



Standard form:

x^2+4x+5

Factored form:

$(x+2+\sqrt{-1})(x+2-\sqrt{-1})$

Vertex form:

$(x--2)^2+1$

$(x+2)^2+1$

$(x+2)(x+2)+1$

$x^2+2x+2x+4+1$

x^2+4x+5

$a=1$
$b=4$
$c=5$

$$x=\frac{-4\pm\sqrt{(4)^2-4\cdot1\cdot5}}{2\cdot1}$$

$$x=\frac{-4\pm\sqrt{16-20}}{2}$$

$$x=\frac{-4\pm\sqrt{-4}}{2}=\frac{-4\pm\sqrt{4}\cdot\sqrt{-1}}{2}=\frac{-4\pm2\sqrt{-1}}{2}$$

$$=-\frac{\cancel{4}^{2}}{\cancel{2}_{1}}\pm\frac{\cancel{2}^{1}\sqrt{-1}}{\cancel{2}_{1}}$$

$$=-2\pm\sqrt{-1}$$

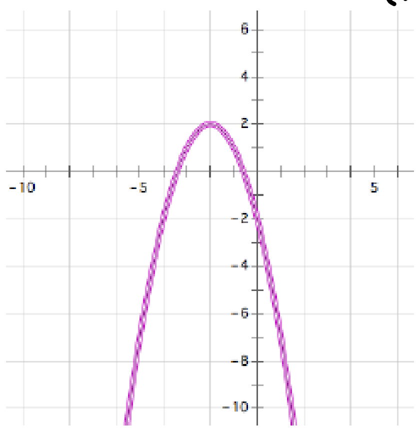
$(x--2+\sqrt{-1})(x--2-\sqrt{-1})$

$(x+2+\sqrt{-1})(x+2-\sqrt{-1})$

Here are some more of Israel and Miriam's homework.

7.

x	y
-5	-7
-4	-2
-3	1
-2	2
-1	1
0	-2
1	-7
2	-14
3	-23
4	-34
5	-47



Standard form:

Factored form:

Vertex form:

The Fundamental Theorem of Algebra

A polynomial function is a function of the form:

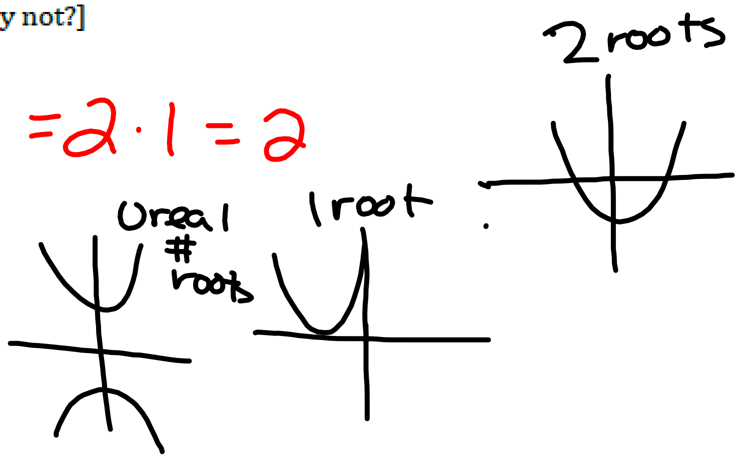
$$y = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$$

where all of the exponents are positive integers and all of the coefficients $a_0 \ldots a_n$ are constants.

As the theory of finding roots of polynomial functions evolved, a 17th century mathematician, Girard (1595-1632) made the following claim which has come to be known as the Fundamental Theorem of Algebra: *An nth degree polynomial function has n roots.*

In later math classes you will study polynomial functions that contain higher-ordered terms such as x^3 or x^5. Based on you work in this task, do you believe this theorem holds for quadratic functions? That is, do all functions of the form $y = ax^2 + bx + c$ always have two roots? [Examine the graphs of each of the quadratic functions you have written equations for in this task. Do they all have two roots? Why or why not?]

$2 = 2x^0 = 2 \cdot 1 = 2$



$5x^3$

x^2

x^2+4x+5

x^2-3x+2

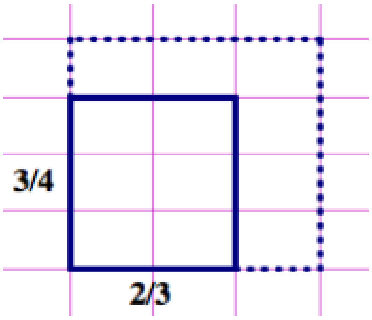
3.9 My Irrational and Imaginary Friends

A Solidify Understanding Task

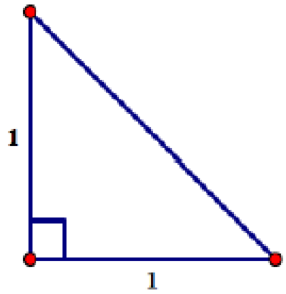
Part 1: Irrational numbers

Find the perimeter of each of the following figures. Express your answer as simply as possible.

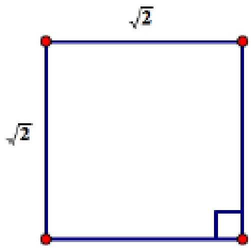
1. The 3/4 x 2/3 rectangle



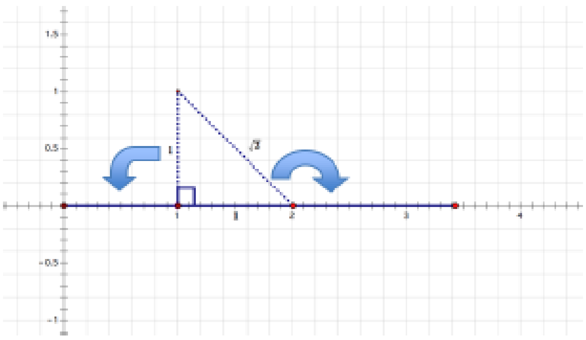
2. The isosceles right triangle



3. The $\sqrt{2}$ x $\sqrt{2}$ square



4. We might approximate the perimeter of figure 2 with a decimal number, but the exact perimeter is $2+\sqrt{2}$, which cannot be simplified any farther. Note that this notation represents a single number—the distance around the perimeter of the triangle—even though it is written as the sum of two terms. We could visualize this single number by laying the three sides of the triangle end-to-end along a number line, starting at 0, so the endpoint of the last segment would be at the number $2+\sqrt{2}$. Is the number we have located on the number line in this way a rational number or an irrational number? Explain your answer.



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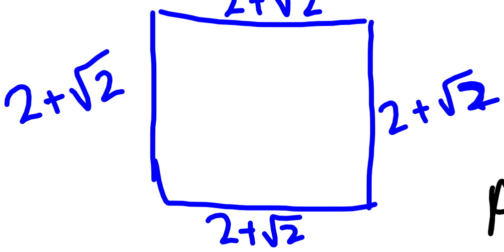
7. Draw a representative image and find the area of the following figures:

(a) a square with sides $2+\sqrt{2}$



Perimeter:

$2+\sqrt{2}+2+\sqrt{2}+2+\sqrt{2}+2+\sqrt{2} =$

$8+4\sqrt{2}$

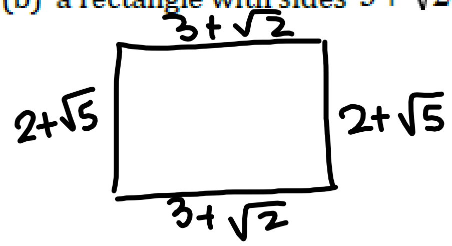
$4(2+\sqrt{2}) = 8+4\sqrt{2}$

Area: $(2+\sqrt{2})(2+\sqrt{2}) =$

$2\cdot 2 + 2\sqrt{2} + 2\sqrt{2} + \sqrt{2}\cdot\sqrt{2} =$

$4 + 4\sqrt{2} + 2 = \underline{6+4\sqrt{2}}$

(b) a rectangle with sides $3+\sqrt{2}$ and $2+\sqrt{5}$



Perimeter:

$2+\sqrt{5}+3+\sqrt{2}+2+\sqrt{5}+3+\sqrt{2} =$

$10+2\sqrt{5}+2\sqrt{2}$

Area:

$(2+\sqrt{5})(3+\sqrt{2}) = 6+2\sqrt{2}+3\sqrt{5}+\sqrt{10}$

(c) a rectangle with sides $3+\sqrt{2}$ and $2+\sqrt{8}$

(d) a rectangle with sides $\sqrt{2}+\sqrt[3]{2}$ and $\sqrt{6}+\sqrt[3]{4}$

Note: The set of numbers that contains all of the *rational numbers* and all of the *irrational numbers* is called the set of *real numbers*.

Part 2: Imaginary and Complex Numbers

In the previous task, you found that the quadratic formula gives the roots of $x^2 + 4x + 5$ as $-2+\sqrt{-1}$ and $-2-\sqrt{-1}$. Because the square root of a negative number has no defined value as either a rational or an irrational number, Euler proposed that a new number $i = \sqrt{-1}$ be including in what came to be known as the complex number system.

$i = \sqrt{-1}$

9. Based on Euler's definition of *i*, what would the value of i^2 be?

$i^2 = \sqrt{-1}^{\,2}$

$i^2 = -1$

With the introduction of the number *i*, the square root of *any* negative number can be represented. For example, $\sqrt{-2} = \sqrt{2}\cdot\sqrt{-1} = \sqrt{2}\cdot i$ and $\sqrt{-9} = \sqrt{9}\cdot\sqrt{-1} = 3i$.

10. Find the values of the following expressions. Show the details of your work.

(a) $\left(\sqrt{2}\cdot i\right)^2 = \sqrt{2}\cdot i\cdot\sqrt{2}\cdot i = \sqrt{4}\cdot i^2 = 2\cdot -1 = -2$

(b) $3i \times 3i = 9i^2 = 9(-1) = -9$

$-2+\sqrt{-1}$ & $-2-\sqrt{-1}$

Using this new notation, the roots of $x^2 + 4x + 5$ can be written as $-2+i$ and $-2-i$, and the factored form of $x^2 + 4x + 5$ can be written as $(x+2-i)(x+2+i)$.

11. Verify that $x^2 + 4x + 5$ and $(x+2-i)(x+2+i)$ are equivalent by expanding and simplifying the factored form. Show the details of your work.

	x	$+2$	$-i$
x	x^2	$2x$	$\cancel{-ix}$
$+2$	$2x$	4	$\cancel{-2i}$
i	$\cancel{xi}$	$\cancel{2i}$	$-i^2$

$x^2+4x+4-i^2$

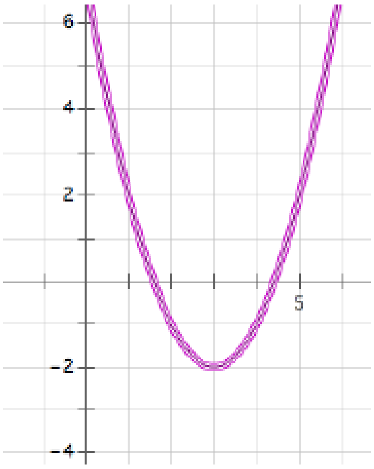
$x^2+4x+4- -1$

x^2+4x+5 ✓

Note: Numbers like $3i$ and $\sqrt{2}\cdot i$ are called *pure imaginary numbers*. Numbers like -2 – *i* and -2 + *i* that include a real term and an imaginary term are called *complex numbers*.

The quadratic formula is usually written in the form $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. An equivalent form is $\frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$. If a, b and c are rational coefficients, then $\frac{-b}{2a}$ is a rational term, and $\frac{\sqrt{b^2 - 4ac}}{2a}$ may be a rational term, an irrational term or an imaginary term, depending on the value of the expression under the square root sign.

12. Examine the roots of the quadratic $y = x^2 - 6x + 7$ shown in the graph at the right. How do the terms $\frac{-b}{2a}$ and $\frac{\sqrt{b^2 - 4ac}}{2a}$ show up in this graph?

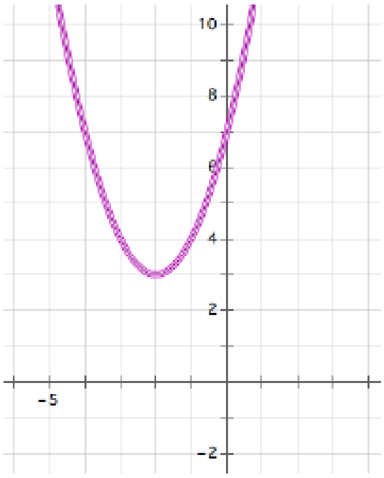


Look back at the work you did in the task *To Be Determined . . .*

13. Which quadratics in that task had complex roots?

14. How can you determine if a quadratic has complex roots from its graph?

15. Find the complex roots of the following quadratic function represented by its graph.



Note: Complex numbers are not real numbers—they do not lie on the real number line that includes all of the rational and irrational numbers; also note that the real numbers are a subset of the complex numbers since a real number results when the imaginary part of $a + bi$ is 0, that is, $a + 0i$.

Homework

3.9 MVP "Ready, Set, Go"