

## Questions on 3.7 and 3.8 HW?

Remainder Theorem (pg.35):

$f(a)$ ,  $a$  is root

$x - a$

$f(a) = 0$ , then  $a$   
is a root of our  
polynomial.

⑨  $f(x) = x^3 + 5x^2 + 2x - 8; f(1)$

$$f(1) = (1)^3 + 5(1)^2 + 2(1) - 8 = 1 + 5 + 2 - 8 = \underline{\underline{0}}$$

Yes, 1 is a root.

$(x-1)$  is a factor.

# Polynomial Division

Just like with whole numbers, we can divide polynomials too!

**Practice.**  
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$$\begin{array}{r} 121 \\ 4 \overline{) 484} \\ \underline{-4} \phantom{0} \\ 08 \phantom{0} \\ \underline{-8} \phantom{0} \\ 04 \phantom{0} \\ \underline{-4} \\ 0 \end{array}$$

$$(3k^4 + 40k^3 + 5k^2) \div 10k^2$$

$$\frac{3k^4}{10k^2} = \frac{3}{10}k^2$$

$$\begin{array}{r} \frac{3}{10}k^2 + 4k + \frac{1}{2} \\ 10k^2 \overline{) 3k^4 + 40k^3 + 5k^2 + 0k + 0} \\ \underline{-(3k^4)} \phantom{0} \\ 0 + 40k^3 \phantom{0} \\ \underline{-(40k^3)} \phantom{0} \\ 0 + 5k^2 \phantom{0} \\ \underline{-(5k^2)} \\ 0 \end{array}$$

Answer:  
 $\frac{3}{10}k^2 + 4k + \frac{1}{2}$

$$(m^3 + m^2 - 28m - 19) \div (m - 5)$$

$$\begin{array}{r} m^2 + 6m + 2 \\ m-5 \overline{) m^3 + m^2 - 28m - 19} \\ \underline{-(m^3 - 5m^2)} \phantom{0} \\ 6m^2 - 28m \phantom{0} \\ \underline{-(6m^2 - 30m)} \phantom{0} \\ 2m - 19 \\ \underline{-(2m - 10)} \\ -9 \end{array}$$

$\frac{6m^2}{m} = 6m$

Answer:  
 $m^2 + 6m + 2 - \frac{9}{m-5}$

$$\frac{3}{2} \rightarrow 2 \overline{) 3.0}$$

**Practice.**

$$(x^3 - x^2 - 32x + 62) \div (x + 6)$$

$$\begin{array}{r} x^2 - 7x + 10 \\ x+6 \overline{) x^3 - x^2 - 32x + 62} \\ \underline{-(x^3 + 6x^2)} \phantom{+ 62} \\ -7x^2 - 32x \phantom{+ 62} \\ \underline{-(-7x^2 - 42x)} \phantom{+ 62} \\ 10x + 62 \end{array}$$

$-x^2 + 6x^2$

Answer:

$$x^2 - 7x + 10 + \frac{2}{x+6}$$

$10x + 62$   
 $\underline{-(10x + 60)}$   
 $\boxed{2}$

$$(b^3 - 5b^2 - 8) \div (b - 5)$$

$$\begin{array}{r} b^2 \\ b-5 \overline{) b^3 - 5b^2 + 0b - 8} \\ \underline{-(b^3 - 5b^2)} \\ 0b^2 + 0b - 8 \end{array}$$

Answer:

$$b^2 - \frac{8}{b-5}$$

or

$$b^2 + 0b - \frac{8}{b-5}$$

$$(3n^3 - 14n^2 - 45n - 37) \div (3n + 4)$$

$$\begin{array}{r} n^2 - 6n - 7 \\ 3n+4 \overline{) 3n^3 - 14n^2 - 45n - 37} \\ \underline{-(3n^3 + 4n^2)} \phantom{- 37} \\ -18n^2 - 45n \phantom{- 37} \\ \underline{-(-18n^2 - 24n)} \phantom{- 37} \\ -21n - 37 \end{array}$$

$-14+4$   
 $-45+124$

Answer:

$$n^2 - 6n - 7 - \frac{9}{3n+4}$$

$-21n - 37$   
 $\underline{-(-21n - 28)}$   
 $\boxed{-9}$

**Practice.**

$$\textcircled{1} (3x^3 - 15x^2 - 13x - 25) \div (x - 6)$$

$$\textcircled{2} (6p^3 - 7p^2 + 5) \div (6p - 7)$$

$$\textcircled{3} (2x^3 - x^2 - 8) \div (2x - 1)$$

$$\textcircled{4} (8a^3 + 7a^2 - 15a + 3) \div (a + 2)$$

$$\textcircled{5} (9n^3 + 10n^2 + 6) \div (9n + 10)$$

$$\textcircled{6} (v^3 - 4v^2 - 29v - 19) \div (v - 8)$$

## Rational Root Theorem:

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0 = 0$$

-Possible rational roots are  $p/q$ , where  $p$  is an integer factor of the constant term ( $a_0$ ) and  $q$  is an integer factor of the leading coefficient ( $a_n$ ).

### Practice.

State the possible rational zeros for each function. Then find all rational zeros.

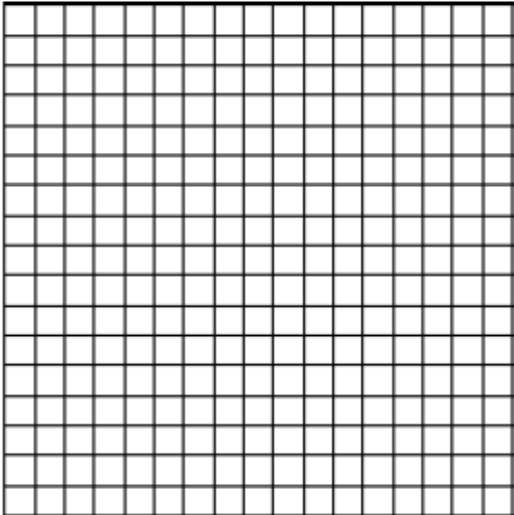
$$f(x) = 9x^3 - 6x^2 + 34x - 11$$

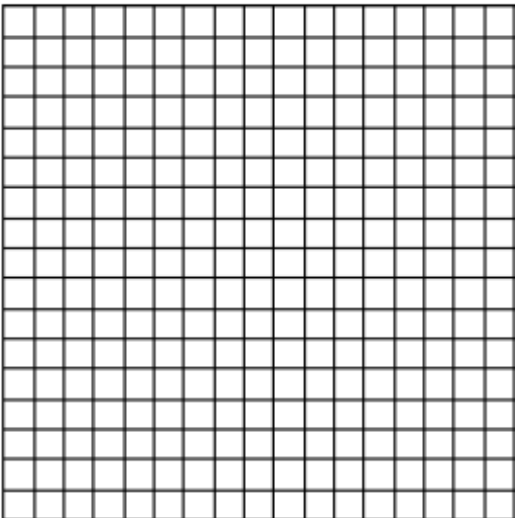
$$f(x) = 2x^3 - x^2 - 2x + 1$$

$$f(x) = x^3 - 5x^2 - 15x + 27$$

$$f(x) = 2x^3 - 5x^2 + 4x - 1$$

From 3.8...turn to page 37

<p>4. <b>Function:</b>  <math>f(x) = 2(x - 1)(x + 3)^2</math></p> <p><b>End behavior:</b>  as <math>x \rightarrow -\infty</math>, <math>f(x) \rightarrow \underline{\hspace{2cm}}</math>  as <math>x \rightarrow \infty</math>, <math>f(x) \rightarrow \underline{\hspace{2cm}}</math></p> <p><b>Roots (with multiplicity):</b></p> <p><b>Value of leading co-efficient:</b></p> <p><b>Domain:</b></p> <p><b>Range:</b> All Real numbers</p>	<p>Graph:</p> 
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<p>5. <b>Function:</b></p> <p><b>End behavior:</b>  as <math>x \rightarrow -\infty</math>, <math>f(x) \rightarrow \infty</math>  as <math>x \rightarrow \infty</math>, <math>f(x) \rightarrow \underline{\hspace{2cm}}</math></p> <p><b>Roots (with multiplicity):</b>  (3,0) m: 1;  (-1,0) m: 2  (0,0) m: 2</p> <p><b>Value of leading co-efficient:</b> -1</p> <p><b>Domain:</b></p> <p><b>Range:</b></p>	<p>Graph:</p> 
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<p>6. <b>Function:</b></p> <p><b>End behavior:</b>  <math>as\ x \rightarrow -\infty, \quad f(x) \rightarrow \underline{\hspace{2cm}}</math>  <math>as\ x \rightarrow \infty, \quad f(x) \rightarrow \underline{\hspace{2cm}}</math></p> <p><b>Roots (with multiplicity):</b></p> <p><b>Value of leading co-efficient: 1</b></p> <p><b>Domain:</b></p> <p><b>Range:</b></p> <p><b>Other:</b> <math>f(-2) = 0</math></p>	<p><b>Graph:</b></p>
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Without using technology, sketch the graph of the polynomial function described. The term “imaginary roots” means complex zeros.

7. A cubic function with a leading coefficient of -2, with two negative zeros and one positive.

8. A quartic function with a leading coefficient of 1, with two negative zeros and one positive double zero.

9. A cubic function with a leading coefficient of -3, with an imaginary root and one positive double root.

10. A quartic function with a leading coefficient of -2, with two negative zeros and one positive double root.

Find all factors and sketch the graph of the polynomial functions.

11.  $f(x) = x^3 - x^2$

12.  $f(x) = x^4 - x^2$

13.  $f(x) = x^3 - 2x$

14.  $f(x) = x^3 - x^2 + 9x - 9$