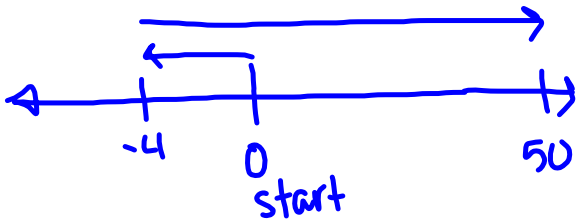
Questions on 3.6 WKS?

(5) e) (0,0)

(2,-4)

(5,50)



58 cm total

g) avg. speed $\frac{58\text{cm}}{5\text{ mins}} = 11.6\text{ cm/min}$

3.7 Trig Review: Derivatives of Trig Functions

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

EXAMPLES

Find the value of each of the following without a calculator.

a. $\cos\left(\sin^{-1}\frac{1}{\sqrt{2}}\right)$

$\cos\left(\sin^{-1}\frac{\sqrt{2}}{2}\right)$

$\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\sin\theta = \frac{\sqrt{2}}{2}$

$\cos\left(\frac{\pi}{4}\right)$

$\frac{\sqrt{2}}{2}$

b. $\sec\left(\tan^{-1}\frac{1}{\sqrt{3}}\right)$

$\sec\left(\tan^{-1}\frac{1}{\sqrt{3}}\right) =$

$\sec\left(\frac{\pi}{6}\right) =$

$\frac{1}{\cos\frac{\pi}{6}} =$

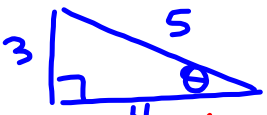
$\frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{2\sqrt{3}}{3}$

$\frac{\pi}{6} \to \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

c. $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$



$\sin\theta = \frac{3}{5}$

$\theta = \sin^{-1}\left(\frac{3}{5}\right)$

$= \frac{5\pi}{4}$

EXAMPLE

Show that $\frac{1-\cos a}{a} = \frac{\sin^2 a}{a(1+\cos a)}$

$\frac{1-\cos^2 a}{a(1+\cos a)} = \frac{(1+\cos a)(1-\cos a)}{a(1+\cos a)} = \frac{(1-\cos a)}{a}$

$\sin^2 u + \cos^2 u = 1$

$\sin^2 u = 1 - \cos^2 u$

$\cos^2 u = 1 - \sin^2 u$

Q.E.D.

Derivative of the Sine Function:

The derivative of the sine is the cosine.

$$\frac{d}{dx}\sin x = \cos x$$

Derivative of the Cosine Function:

The derivative of the cosine is the negative of the sine.

$$\frac{d}{dx}\cos x = -\sin x$$

$(\sin(x))^2 = \sin^2 x = (\sin x)(\sin x)$

Find derivative of $\frac{\sin x}{(\cos x - 2)}$

$$\frac{dy}{dx} = \frac{(\cos x - 2)(\cos x) - \sin x(-\sin x)}{(\cos x - 2)^2} = \frac{\cos^2 x - 2\cos x + \sin^2 x}{(\cos x - 2)^2} = 1 - \frac{2\cos x}{(\cos x - 2)^2}$$

use $\sin^2 x + \cos^2 x = 1$

OR $\frac{1 - 2\cos x}{\cos^2 x - 4\cos x + 4}$

Example:

Find the derivative of $\dfrac{\sin x}{(\cos x - 2)}$.

$$\frac{dy}{dx} = \frac{(\cos x - 2)\frac{d}{dx}\sin x - \sin x \frac{d}{dx}(\cos x - 2)}{(\cos x - 2)^2} \quad \text{quotient rule}$$

$$= \frac{(\cos x - 2)(\cos x) - \sin x(-\sin x)}{(\cos x - 2)^2}$$

$$= \frac{\cos^2 x - 2\cos x + \sin^2 x}{(\cos x - 2)^2}$$

$$= \frac{(\sin^2 x + \cos^2 x) - 2\cos x}{(\cos x - 2)^2} \quad \sin^2 x + \cos^2 x = 1$$

$$= \frac{1 - 2\cos x}{(\cos x - 2)^2}$$

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Derivatives of the Other Basic Trigonometric Functions:

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

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EXAMPLES
No calculator
Find y' if

a. $y = x^2 \sin x$ (product rule)

$y' = (x^2)(\cos x) + (\sin x)(2x)$

$y' = x^2 \cos x + 2x \sin x$

b. $y = \dfrac{\cos x}{1 - \sin x}$ (quotient rule)

$y' = \dfrac{(1-\sin x)(-\sin x) - (\cos x)(-\cos x)}{(1-\sin x)^2}$ (0 − cos x)

$= \dfrac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2}$ ($\sin^2 x + \cos^2 x = 1$)

$= \dfrac{1 - \sin x}{(1-\sin x)^2} = \dfrac{1}{1-\sin x}$

Find y'' if $y = \sec x$.

$y' = \sec x \tan x$

$y'' = \sec x(\sec^2 x) + \tan x(\sec x \tan x)$

$y'' = \sec^3 x + \sec x \tan^2 x$ OR $y'' = \sec x(\sec^2 x + \tan^2 x)$

$-\sec x(-\sec^2 x - \tan^2 x)$

$1 + \tan^2 x = \sec^2 x$

$1 = \sec^2 x - \tan^2 x$

A weight hanging from a spring is stretched 5 units beyond its rest position ($s = 0$) and released at time $t = 0$ to bob up and down. Its position at any later time t is given by the function $s = 5\cos t$. What are its velocity and acceleration at time t?

$s = 5\cos t$

$v = s' = 5(-\sin t) = -5\sin t$

$a = v' = s'' = -5\cos t$

Calculator

Find the equations of the lines that are tangent and normal to the curve $f(x)=\frac{\tan x}{x}$ at $x=2$.

$f'(x)=\frac{x(\sec^2 x)-(\tan x)(1)}{x^2}=\frac{x\sec^2 x-\tan x}{x^2}$

$f'(2)=\frac{2(\sec^2(2)-\tan(2))}{2^2}=3.43$

$f(2)=\frac{\tan(2)}{2}=-1.09 \rightarrow (2,-1.09)$

tangent: $y+1.09=3.43(x-2)$

$y=3.43x-7.93$

normal: $m=-\frac{1}{3.43}$

$y+1.09=\frac{-1}{3.43}(x-2)$

$y=-0.292x-0.507$

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Example:

Find the equation of a line tangent to $y = x\cos x$ at $x = 1$.

$y = x\cos x$

$m = \frac{d}{dx}(x\cos x) = x(-\sin x) + \cos x(1)$

Evaluate m when $x = 1$

$m = 1(-.8414709848) + (.5403023059) = -.3011686789$

$m = 1(-.8414709848) + (.5403023059) = -.3011686789$

When $x = 1$, $y = 1(\cos 1) = .5403023059$

The equation of the tangent line is

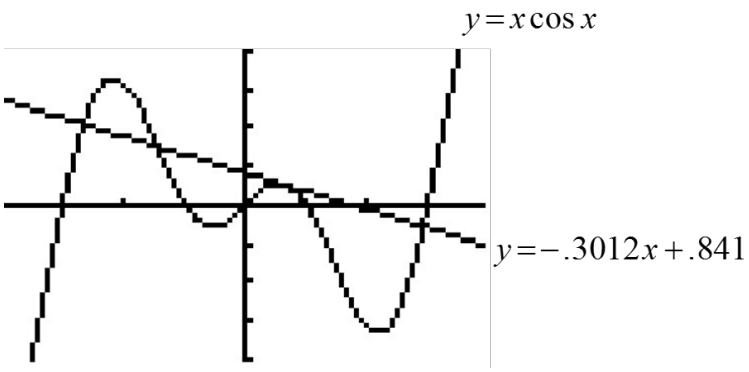
$y - .5403023059 = -.3011686789(x - 1)$

$y = -.3011686789x + .3011686789 + .5403023059$

$y = -.3011686789x + .8414709848$

After rounding the equation is

$y = -.3012x + .841$



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Homework

3.7 pg.146-147 #3-33odds, 30, 37

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