

Questions on 3.5 HW?

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While working with areas it sometimes essential to convert between units of measure, for example changing from square yards to square feet and so forth. Convert the areas below to the desired measure. (Hint: area is two dimensional, for example $1 \text{ yd}^2 = 9 \text{ ft}^2$ because 3 ft along each side of a square yard equals 9 square feet.)

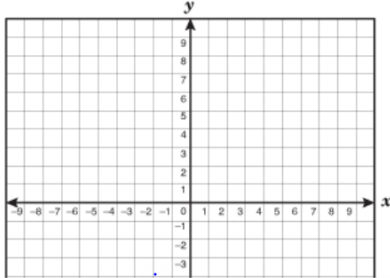
1. $7 \text{ yd}^2 = ? \text{ ft}^2$ 2. $5 \text{ ft}^2 = ? \text{ in}^2$ 3. $1 \text{ mile}^2 = ? \text{ ft}^2$

4. $100 \text{ m}^2 = ? \text{ cm}^2$ 5. $300 \text{ ft}^2 = ? \text{ yd}^2$ 6. $96 \text{ in}^2 = ? \text{ ft}^2$

Set
 Topic: Transformations and Parabolas, Symmetry and Parabolas

7a. Graph each of the quadratic functions.
 $f(x) = x^2$
 $g(x) = x^2 - 9$
 $h(x) = (x + 2)^2 - 9$

b. How do the functions compare to each other?



Handwritten notes and calculations:

$1 \text{ yd} = 3 \text{ ft}$
 $A = 1 \text{ yd}^2$
 $A = 9 \text{ ft}^2$
 $1 \text{ yd}^2 = 9 \text{ ft}^2$

$1 \text{ mi} = 5280 \text{ ft}$
 $A = 1 \text{ mi}^2$
 $A = 27,878,400 \text{ ft}^2$
 $1 \text{ mi}^2 = 27,878,400 \text{ ft}^2$

$\frac{1 \text{ yd}^2}{9 \text{ ft}^2} \text{ or } \frac{9 \text{ ft}^2}{1 \text{ yd}^2}$

$\frac{1 \text{ mi}^2}{27,878,400 \text{ ft}^2} \text{ or } \frac{27,878,400 \text{ ft}^2}{1 \text{ mi}^2}$

(1) $7 \text{ yd}^2 = 63 \text{ ft}^2$

$\frac{3 \text{ mi}^2}{1} \cdot \frac{27,878,400 \text{ ft}^2}{1 \text{ mi}^2} =$

10. $f(x) = x^2 + 4x - 12 = (x+6)(x-2)$

a. $f(0) = \underline{\hspace{2cm}}$

b. $f(2) = \underline{\hspace{2cm}}$

c. $f(x) = 0, \quad x = \underline{-6, 2} \quad 0 = (x+6)(x-2)$
 $x = -6, 2$

d. $f(x) = 20, \quad x = \underline{\hspace{2cm}}$

$\frac{1 \text{ m}^2}{10,000 \text{ cm}^2} \text{ or } \frac{10,000 \text{ cm}^2}{1 \text{ m}^2}$

10. $f(x) = x^2 + 4x - 12 = (x+6)(x-2)$

a. $f(0) = \frac{-12}{(0, -12)}$ $f(0) = 0^2 + 4 \cdot 0 - 12 = -12$

b. $f(2) = \frac{0}{(2, 0)}$ $f(2) = (2+6)(2-2) = 8 \cdot 0 = 0$

c. $f(x) = 0$, $x = \frac{-6, 2}{(-6, 0), (2, 0)}$ $0 = (x+6)(x-2)$
 $x = -6, 2$

d. $f(x) = 20$, $x = \frac{-8, 4}{(-8, 0), (4, 0)}$

$20 = x^2 + 4x - 12$
 $-20 \quad -20$

$0 = x^2 + 4x - 32$

$0 = (x+8)(x-4)$

$x = -8, 4$

11) $g(x) = (x-5)^2 + 2$

a) $g(0) = (0-5)^2 + 2 = \square$

b) $g(5) = (5-5)^2 + 2 = \square$

c) $g(x) = 0$, $x = \text{no solution}$ $0 = (x-5)^2 + 2$
 $-2 \quad -2$

d) $g(x) = 16$, $x = \underline{\hspace{2cm}}$
 $16 = (x-5)^2 + 2$
 $\sqrt{-2} = \sqrt{(x-5)^2}$
 $\sqrt{-2} = x-5$
no solution

$\sqrt{14} = \sqrt{(x-5)^2}$

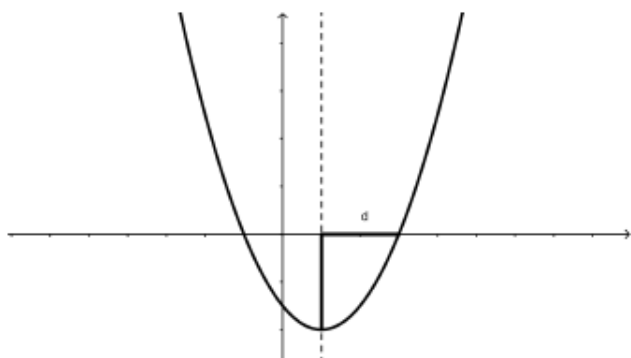
$\pm\sqrt{14} = x - 5$
 $+5 \quad +5$

$5 \pm \sqrt{14} = x$

$x = 5 + \sqrt{14}, 5 - \sqrt{14}$

$x = \underline{\hspace{2cm}}$

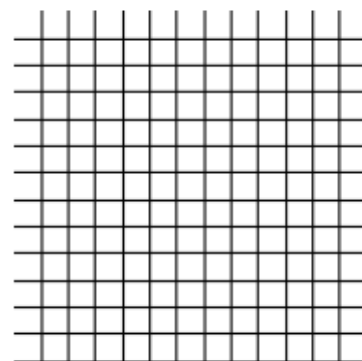
Just to make it a little easier to talk about some of the features that relate to the intercepts, let's name them with variables. From now on, when we talk about the distance from the line of symmetry to either of the x intercepts, we'll call it d . The diagram below shows this feature.



We will always refer to the line of symmetry as the line $x = h$, so the two x -intercepts will be at the points $(h - d, 0)$ and $(h + d, 0)$.

3. So, let's think about another function: $f(x) = x^2 - 6x + 4$

- Graph the function by putting the equation into vertex form.
- What is the vertex of the function?
- What is the equation of the line of symmetry?
- What do you estimate the x -intercepts of the function to be?
- What do you estimate d to be?
- What is the value of $f(x)$ at the x -intercepts?
- Using the vertex form of the equation and your answer to "f" above, write an equation and solve it to find the exact values of the x intercepts.
- What is the exact value of d ?
- Use a calculator to find approximations for the x -intercepts. How do they compare with your estimates?



4. What about a function with a vertical stretch? Can we find exact values for the x-intercepts the same way? Let's try it with: $f(x) = 2x^2 - 8x + 5$.

a. Graph the function by putting the equation into vertex form.

b. What is the vertex of the function?
 $(2, -3)$

c. What is the equation of the line of symmetry?
 $x = 2$

d. What do you estimate the x-intercepts of the function to be?
 $x = 0.8, 3.1$ or $(0.8, 0)$
 $(3.1, 0)$

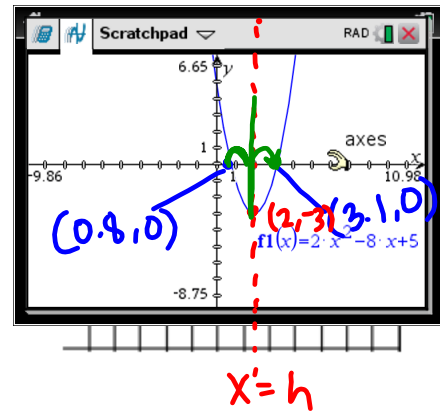
e. What do you estimate d to be?

f. What is the value of $f(x)$ at the x-intercepts?
 $f(x) = 0$

g. Using the vertex form of the equation and your answer to "f" above, write an equation and solve it to find the exact values of the x-intercepts.

h. What is the exact value of d ?

i. Compare your solution to your estimate of the roots. How did you do?



axis of symmetry: $x = \frac{-b}{2a}$ { Standard form $ax^2 + bx + c$

$f(x) = 2x^2 - 8x + 5$

$a = 2$
 $b = -8$
 $c = 5$

Axis of Symm.: $x = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$
 $x = 2$

Quadratic Formula

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 2 \cdot 5}}{2 \cdot 2}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{8 \pm \sqrt{64 - 40}}{4}$

$= \frac{8 \pm \sqrt{24}}{4}$

$\rightarrow \frac{8 + \sqrt{24}}{4} = 3.2$

$\rightarrow \frac{8 - \sqrt{24}}{4} = 0.8$

6. How could you use the solutions you just found to tell what the x -intercepts are for the function $f(x) = x^2 - 3x - 1$?
7. You may have found the algebra for writing the general quadratic function $f(x) = ax^2 + bx + c$ in vertex form a bit difficult. Here is another way we can work with the general quadratic function leading to the same results you should have arrived at in 5d.
- Since the two x -intercepts are d units from the line of symmetry $x = h$, if the quadratic crosses the x -axis its x -intercepts are at $(h - d, 0)$ and $(h + d, 0)$. We can always write the factored form of a quadratic if we know its x -intercepts. The factored form will look like $f(x) = a(x - p)(x - q)$ where p and q are the two x -intercepts. So, using this information, write the factored form of the general quadratic $f(x) = ax^2 + bx + c$ using the fact that its x -intercepts are at $h - d$ and $h + d$.
 - Multiply out the factored form (you will be multiplying two **trinomial** expressions together) to get the quadratic in standard form. Simplify your result as much as possible by combining like terms.
 - You now have the same general quadratic function written in standard form in two different ways, one where the **coefficients** of the terms are a , b and c and one where the coefficients of the terms are expressions involving a , h and d . Match up the coefficients; that is, b , the coefficient of x in one version of the standard form is equivalent to _____ in the other version of the standard form. Likewise c , the constant term in one version of the standard form is equivalent to _____ in the other.
 - Solve the equations $b = \underline{\hspace{2cm}}$ and $c = \underline{\hspace{2cm}}$ for h and d . Work with your equations until you can express h and d with expressions that only involve a , b and c .
 - Based on this work, how can you find the x -intercepts of any quadratic using only the values for a , b and c ?
 - How does your answer to "e" compare to your result in 5d?
8. All of the functions that we have worked with on this task have had graphs that open upward. Would the formula work for parabolas that open downward? Tell why or why not using an example that you create using your own values for the coefficients a , b , and c .

3.6 Curbside Rivalry

A Solidify Understanding Task

Carlos and Clarita have a brilliant idea for how they will earn money this summer. Since the community in which they live includes many high schools, a couple of universities, and even some professional sports teams, it seems that everyone has a favorite team they like to root for. In Carlos' and Clarita's neighborhood these rivalries take on special meaning, since many of the neighbors support different teams. They have observed that their neighbors often display handmade posters and other items to make their support of their favorite team known. The twins believe they can get people in the neighborhood to buy into their new project: painting team logos on curbs or driveways.



For a small fee, Carlos and Clarita will paint the logo of a team on a neighbor's curb, next to their house number. For a larger fee, the twins will paint a mascot on the driveway. Carlos and Clarita have designed stencils to make the painting easier and they have priced the cost of supplies. They have also surveyed neighbors to get a sense of how many people in the community might be interested in purchasing their service. Here is what they have decided, based on their research.

- *A curbside logo will require 48 in² of paint*
- *A driveway mascot will require 16 ft² of paint*
- *Surveys show the twins can sell 100 driveway mascots at a cost of \$20, and they will sell 10 fewer mascots for each additional \$5 they charge*

1. If a curbside logo is designed in the shape of a square, what will its dimensions be?

A square logo will not fit nicely on a curb, so Carlos and Clarita are experimenting with different types of rectangles. They are using a software application that allows them to stretch or shrink their logo designs to fit different rectangular dimensions.

2. Carlos likes the look of the logo when the rectangle in which it fits is 8 inches longer than it is wide. What would the dimensions of the curbside logo need to be to fit in this type of rectangle? As part of your work, write a quadratic equation that represents these requirements.

- Clarita prefers the look of the logo when the rectangle in which it fits is 13 inches longer than it is wide. What would the dimensions of the curbside logo need to be to fit in this type of rectangle? As part of your work, write a quadratic equation that represents these requirements.

Your quadratic equations on the previous two problems probably started out looking like this: $x(x + n) = 48$ where n represents the number of inches the rectangle is longer than it is wide. The expression on the left of the equation could be multiplied out to get an equation of the form $x^2 + nx = 48$. If we subtract 48 from both sides of this equation we get $x^2 + nx - 48 = 0$. In this form, the expression on the left looks more like the quadratic functions you have been working with in previous tasks, $y = x^2 + nx - 48$.

- Consider Carlos' quadratic equation where $n = 8$, so $x^2 + 8x - 48 = 0$. How can we use our work with quadratic functions like $y = x^2 + 8x - 48$ to help us solve the quadratic equation $x^2 + 8x - 48 = 0$? Describe at least two different strategies you might use, and then carry them out. Your strategies should give you solutions to the equation as well as a solution to the question Carlos is trying to answer in #2.
- Now consider Clarita's quadratic equation where $n = 13$, so $x^2 + 13x - 48 = 0$. Describe at least two different strategies you might use to solve this equation, and then carry them out. Your strategies should give you solutions to the equation as well as a solution to the question Clarita is trying to answer in #3.
- After much disagreement, Carlos and Clarita agree to design the curbside logo to fit in a rectangle that is 6 inches longer than it is wide. What would the dimensions of the curbside logo need to be to fit in this type of rectangle? As part of your work, write and solve a quadratic equation that represents these requirements.

7. What are the dimensions of a driveway mascot if it is designed to fit in a rectangle that is 6 feet longer than it is wide? (See the requirements for a driveway mascot given in the bulleted list above.) As part of your work, write and solve a quadratic equation that represents these requirements.

8. What are the dimensions of a driveway mascot if it is designed to fit in a rectangle that is 10 feet longer than it is wide? (See the requirements for a driveway mascot given in the bulleted list above.) As part of your work, write and solve a quadratic equation that represents these requirements.

Carlos and Clarita are also examining the results of their neighborhood survey, trying to determine how much they should charge for a driveway mascot. Remember, this is what they have found from the survey: *They can sell 100 driveway mascots at a cost of \$20, and they will sell 10 fewer mascots for each additional \$5 they charge.*

9. Write an equation, make a table, and sketch a graph for the number of driveway mascots the twins can sell for each \$5 increment, x , in the price of the mascot.

10. Write an equation, make a table, and sketch a graph (on the same set of axes) for the price of the driveway mascot for each \$5 increment, x , in the price.

11. Write an equation, make a table, and sketch a graph for the revenue the twins will collect for each \$5 increment in the price of the mascot.

12. The twins estimate that the cost of supplies will be \$250 and they would like to make \$2000 in profit from selling driveway mascots. Therefore, they will need to collect \$2250 in revenue. Write and solve a quadratic equation that represents collecting \$2250 in revenue. Be sure to clearly show your strategy for solving this quadratic equation.

Homework

Finish 3.6 "Ready, Set, Go"