Questions on 3.4 HW {pg.124-5 #25-42all}?

M.C. - 1.2 pts
F.R - 9 pts

37) $y = x^3 - 3x + 1$; $(2,3)$

$y' = 3x^2 - 3$

$y'(2) = 3(2)^2 - 3$

$y'(2) = 9 \longrightarrow \perp$ line: $-\frac{1}{9}$

$y - 3 = -\frac{1}{9}(x-2)$

$y = -\frac{1}{9}x + \frac{2}{9} + 3$

$\boxed{y = -\frac{1}{9}x + \frac{29}{9}}$

38) $y = x^3 + x$

$y' = 3x^2 + 1$

$4 = 3x^2 + 1$
$-1 \qquad -1$

$\frac{3}{3} = \frac{\cancel{3}x^2}{\cancel{3}}$

$\sqrt{1} = \sqrt{x^2}$

$\pm 1 = x$

$x = 1 \longrightarrow 1^3 + 1 = 2$
* $(1,2)$

$x = -1 \longrightarrow (-1)^3 + (-1) = -2$
* $(-1,-2)$

slope = 4

$y - 2 = 4(x-1)$

$y = 4x - 4 + 2$

$\boxed{y = 4x - 2}$

$y + 2 = 4(x+1)$

$y = 4x + 4 - 2$

$\boxed{y = 4x + 2}$

Oct 6-7:43 AM

3.5 Derivatives as Rates of Change; Position, Velocity, Acceleration

Instantaneous Rate of Change

The (instantaneous) rate of change of f with respect to x at a is the derivative

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

provided the limit exists.

When we say rate of change, we mean instantaneous rate of change.

Example:

If the area of a square as a function of the radius is $A=\pi r^2$,

(a) Find the rate of change of the area A with respect to the radius r.

(b) Evaluate the rate of change of A when $r=4$.

$A(r)=\pi r^2$

$A'(r)=\pi\cdot 2r$

$A'(r)=2\pi r$

$A'(4)=2\pi(4)$

$A'(4)=8\pi$

(a) The rate of change is the derivative $\dfrac{dA}{dr}=2\pi r$

(b) The rate of change when $r=4$ is $2\pi(4)=8\pi$

Oct 12-2:56 PM

Motion Along a Line

Suppose that an object is moving along a coordinate line so that we know its position s on that line as a function of time t:

$s = f(t)$

The **displacement** of the object over the time interval from t to $t + \Delta t$ is $\Delta s = f(t + \Delta t) - f(t)$.

Like the change in y

The **average velocity** of the object over that time interval is

$$v_{av} = \frac{\text{displacement}}{\text{travel time}} = \frac{\Delta s}{\Delta t} = \frac{f(t+\Delta t) - f(t)}{\Delta t}.$$

Instantaneous Velocity

The (instantaneous) velocity is the derivative of the position function $s = f(t)$ with respect to time. At time t the velocity is

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}.$$

Speed

Speed is the absolute value of velocity.

$$\text{Speed } = |v(t)| = \left|\frac{ds}{dt}\right|$$

Acceleration

Acceleration is the derivative of velocity with respect to time.

If a body's velocity at time t is $v(t) = \frac{ds}{dt}$ then the body's acceleration at time t is $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$.

(function) position : $s = f(t)$ $\quad s$

(1st derivative) velocity : $\frac{ds}{dt}$ $\quad s' = v$

(2nd derivative) acceleration: $\frac{d^2s}{ds^2}$ $\quad s'' = v' = a$

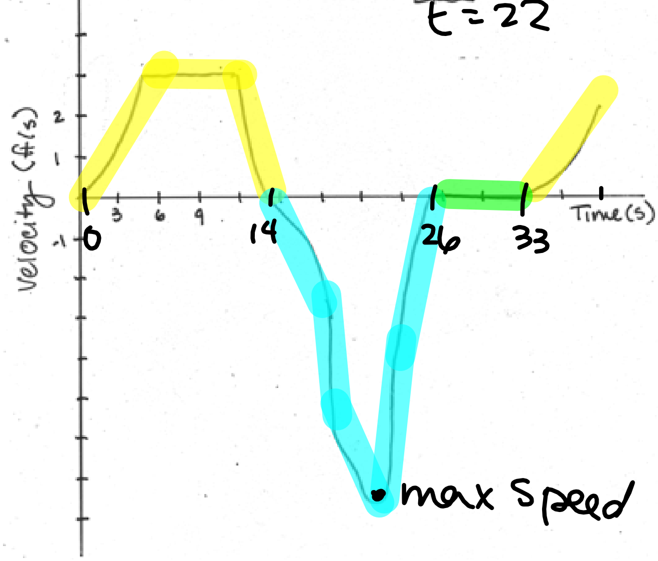
1. The graph below shows the velocity (in ft/s) of a woman walking.

When is she moving forward? $0 < t < 14$ & $33 < t < 39$

When is she moving backward? $14 < t < 26$

When is she standing still? $t = 0, 14, 26, 33$; $26 < t < 33$

When is her speed a maximum? $t = 22$



Velocity:
+ moving forward
− moving backward

2. A dynamite blast propels a heavy rock straight up with a launch velocity of 160 ft/s. Its height is given by the function $s = 160t - 16t^2$, where t is measured in seconds.

a. How high did the rock go?

$s = 16t(10 - t)$

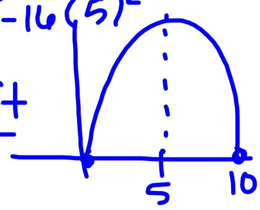
$t = 0, 10$

$0 = 16t(10 - t)$

max: $t = 5$

$s(5) = 160(5) - 16(5)^2$

$s(t) = \underline{400 \text{ ft}}$



b. What is the velocity and speed of the rock when it is ~~256 ft~~ height above the ground on the way up? On the way down?

$s' = v(t) = 160 - 32t$

$v(2) = 160 - 32(2)$

$v(2) = 96$

$v(8) = 160 - 32(8)$

$v(8) = -96$

Speed $= 96$ ft/s

$256 = 160t - 16t^2$

$16t^2 - 160t + 256 = 0$

$16(t^2 - 10t + 16) = 0$

$16(t - 8)(t - 2) = 0$

$t = 8, 2$ sec.

c. What is the acceleration of the rock at any time during its flight?

$s'' = v'(t) = a(t) = -32 \text{ ft/s}^2$

d. When does the rock hit the ground?

$t = 10$ sec

Examples:

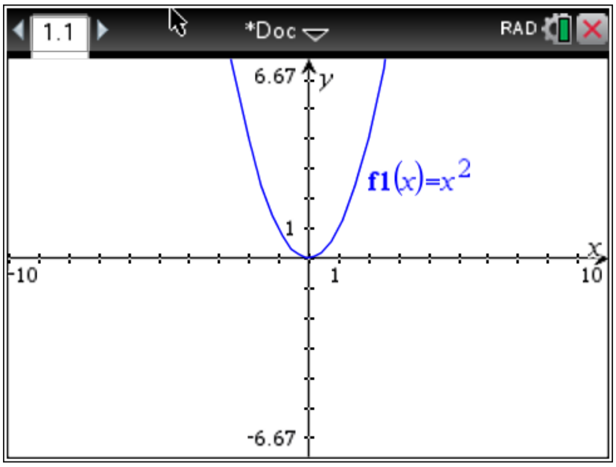
A projectile is shot upward from the surface of the earth and reaches a height of $s = -4.9t^2 + 120t$ meters after t seconds.
Find the velocity after 5 seconds.

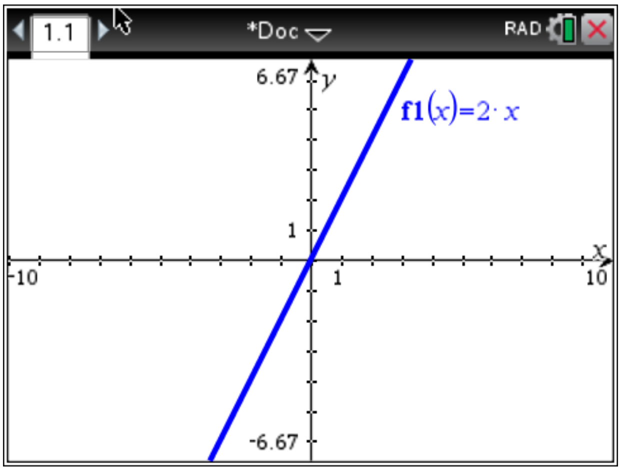
[redacted]

The velocity in ft/s of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$.
What is the acceleration of the particle at $t = 2.63$ s?

Let f be the function given by $f(x) = 2e^{4x^2}$. What is the slope of the tangent to the graph at $x = -1.362$?

Here's a f(x)..........................and its f'(x)





Comparing f(x) and f'(x)...

f(x)	f'(x)

Graph Match Activity

From last time...

6. Let $f(x) = 4x^3 - 3x - 1$. An equation of the line tangent to $y = f(x)$ at $x = 2$ is

(A) $y = 25x - 5$

(B) $y = 45x + 65$

(C) $y = 45x - 65$

(D) $y = 65 - 45x$

(E) $y = 65x - 45$

7. What are the equations of the lines tangent to the graph of $y = x^2 + x$ at $y = 12$?

8. At what point on the graph of $y = \frac{1}{4}x^4$ is the tangent line parallel to the line $x - 8y = 16$?

Homework

3.5 pg.135-137 #3-30 (X3)