Questions on 3.2 HW? Also we will have a quiz on Friday this week on what we've covered so far in unit 3.

(1) left: $y=x^2$

$$\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}=$$

$$\lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}}=\lim_{h\to 0}2x+h=2x+0=\boxed{2x}$$

$$f'(0)=2(0)=0$$

right: $y=x$

$$\lim_{h\to 0}\frac{\cancel{x}+h-\cancel{x}}{h}=\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}}=\boxed{1}$$

$$f'(0)=1$$

$0\neq 1$, so not diff. at (0,0); diff. everywhere else.

(39) $f(x)=\begin{cases}3-x, & x<1\\ ax^2+bx, & x\geq 1\end{cases}$ a & b are constants

a) $\lim_{x\to 1^-}f(x)=f(1)$

$$\lim_{x\to 1^-}(3-x)=3-1=a(1)^2+b(1)$$

also assume $f(1)=2$

$$\boxed{2=a+b}$$

~~$b=2-a$~~

$a=2-b$

(b) left ~~limit~~ derivative: $\lim_{h\to 0^-}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0^-}\frac{(3-(1+h))-2}{h}=$

$$\lim_{h\to 0^-}\frac{\cancel{2}-h-\cancel{2}}{h}=\lim_{h\to 0^-}\frac{-h}{h}=-1$$

right limit/derivative: $\lim_{h\to 0^+}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0^+}\frac{a(1+h)^2+b(1+h)-2}{h}=$

$a+b=2$

$a+b-2=0$

$$\lim_{h\to 0^+}\frac{a(h^2+2h+1)+b+bh-2}{h}=$$

$$\lim_{h\to 0^+}\frac{ah^2+2ah+a+b+bh-2}{h}=$$

$$\lim_{h\to 0^+}\frac{(ah^2+2ah+bh)+\cancel{(a+b-2)}}{h}=\lim_{h\to 0^+}\frac{\cancel{h}(ah+2a+b)}{\cancel{h}}=$$

$2-2=0$

$$\lim_{h\to 0^+}ah+2a+b=a(0)+2a+b=\boxed{2a+b}$$

$\Rightarrow 2a+b=-1$

$b=2-a$

$2a+2-a=-1$

$a+2=-1$

$\boxed{a=-3}$

$b=2--3$

$\boxed{b=5}$

$\begin{cases}3-x\\ -3x^2+5x\end{cases}$

3.3 Rules for Differentiation

1. Derivative of a Constant Function

If f is the function with the constant value c, then

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0$$

This means that the derivative of every constant function is the zero function.

EXAMPLES

$y = 7$ $\frac{dy}{dx} = 0$

$f(x) = 0$ $f'(x) = 0$

$s(t) = -\sqrt{3}$ $\frac{ds}{dt} = 0$

$y = 8\pi^2$ $y' = 0$

2a. Power Rule for Positive Integer Powers of x.

If n is a positive integer, then

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

The Power Rule says:

To differentiate x^n, multiply by n and subtract 1 from the exponent.

2b. Power Rule for Negative Integer Powers of x.

If n is a negative integer and $x \neq 0$, then

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}.$$

This is basically the same as Rule 2 except now n is negative.

EXAMPLES

If $f(x)=x^2$ then $f'= 2\cdot x^{2-1} = 2x$

If $y=x^7$ then $\frac{dy}{dx}= 7\cdot x^{7-1} = 7x^6$

If $g(x)=\sqrt[3]{x}$ then $g'(x)= \frac{1}{3}x^{1/3-1} = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}} = \frac{1}{3\sqrt[3]{x^2}}$

$x^{1/3}$

If $y=\frac{1}{x^5}$ then $y'= -5x^{-5-1} = -5x^{-6} = -\frac{5}{x^6}$

x^{-5}

If $A(r)=\frac{1}{\sqrt[3]{r^3}}$ then $\frac{dA}{dr}= -1\cdot r^{-1-1} = -r^{-2} = -\frac{1}{r^2}$

$\frac{1}{\sqrt[3]{r^3}} = \frac{1}{r} = r^{-1}$

3. The Constant Multiple Rule

If u is a differentiable function of x and c is a constant, then

$$\frac{d}{dx}(cu) = c\frac{du}{dx}$$

This says that if a differentiable function is multiplied by a constant, then its derivative is multiplied by the same constant.

EXAMPLES

If $f(x) = -5x^5$ then $f'(x) = -5 \cdot 5x^{5-1}$

$-5\frac{df}{dx}x^5$ $= -25x^4$

If $y = \frac{3}{x^2}$ then $y' = -\frac{6}{x^3}$

If $f(t) = \frac{4t^2}{5}$ then $\frac{df}{dt} = \frac{8}{5}t$

If $f(x) = \frac{-3x}{2}$ then $f' = -\frac{3}{2}$

If $y = 2\sqrt{x}$ then $\frac{dy}{dx} = 2 \cdot \frac{1}{2}x^{\frac{1}{2}-1} = 1x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$

$2\frac{dy}{dx}x^{\frac{1}{2}}$

4. The Sum and Difference Rule

If u and v are differentiable functions of x, then their sum and differences are differentiable at every point where u and v are differentiable.
At such points,

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}.$$

EXAMPLES: Find the derivative of each of the following.

$\sqrt{x} = x^{1/2}$

$y = 5x^2 - \sqrt{2}x^3 - \sqrt{x}$

$y' = 10x - 3\sqrt{2}\,x^2 - \frac{1}{2}x^{-1/2} =$

$10x - 3\sqrt{2}\,x^2 - \frac{1}{2\sqrt{x}}$

$g(x) = -\frac{x^4}{2} + \frac{3x}{5} - \frac{7}{x}$

$= -\frac{1}{2}x^4 + \frac{3}{5}x - 7x^{-1}$

$g'(x) = -2x^3 + \frac{3}{5} + 7x^{-2}$

$= -2x^3 + \frac{3}{5} + \frac{7}{x^2}$

Does $y = x^4 - 2x^2 + 2$ have any horizontal tangents? If so, where do they occur?

$y' = 4x^3 - 4x$

$y' = 4x(x^2 - 1)$

$0 = 4x(x+1)(x-1)$

horizontal tangent lines at $x = 0, -1, 1$

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5. The Product Rule

The product of two differentiable functions u and v is differentiable, and

$$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx} \quad = uv' + vu'$$

The derivative of a product is actually the sum of two products.

EXAMPLES: Find the derivative of each of the following functions.

$$y=\underbrace{(x^2+3x+5)}_{u}\underbrace{(2x^5+7x-11)}_{v}$$

$$y' = \underbrace{(x^2+3x+5)}_{u}\underbrace{(10x^4+7)}_{\frac{dv}{dx} \text{ or } v'} + \underbrace{(2x^5+7x-11)}_{v}\underbrace{(2x+3)}_{u' \text{ or } \frac{du}{dx}}$$

$$= 10x^6 + 7x^2 + 30x^5 + 21x + 50x^4 + 35 +$$
$$4x^6 + 14x^2 + 6x^5 + 21x \quad -33 - 22x$$
$$14x^6 + 21x^2 + 36x^5 + 20x + 50x^4 + 2$$

$$y' = 14x^6 + 36x^5 + 50x^4 + 21x^2 + 20x + 2$$

$$g(x)=(x^2+1)\left(5-\frac{2}{\sqrt{x}}\right)$$

If $y=uv$ find $y'(2)$ if $u(2)=3$, $u'(2)=-4$, $v(2)=1$, and $v'(2)=2$.

6. The Quotient Rule

At a point where $v \neq 0$, the quotient $y = \frac{u}{v}$ of two differentiable functions is differentiable, and

$$\frac{low \; d \cdot high - high \; d \; low}{low^2}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Since order is important in subtraction, be sure to set up the numerator of the Quotient rule correctly.

EXAMPLES: Find the derivative of each of the following.

$$f(x) = \frac{\overset{u}{x^2 - 1}}{\underset{v}{x^2 + 1}}$$

$$f'(x) = \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2} =$$

$$\frac{\cancel{2x^3} + 2x - \cancel{2x^3} + 2x}{(x^2+1)^2} = \boxed{\frac{4x}{(x^2+1)^2}}$$

$$f(x) = \frac{3 - \frac{1}{x}}{x + 5}$$

$$y = \frac{(x-1)(x^2+2)}{x^3}$$

Second and Higher Order Derivatives

The multiple-prime notation begins to lose its usefulness after three primes.

So we use $y^{(n)} = \frac{d}{dx} y^{(n-1)}$ "y super n"

to denote the nth derivative of y with respect to x.

Do not confuse the notation $y^{(n)}$ with the nth power of y, which is y^n.

Oct 11-2:08 PM

Homework

Derivatives Worksheet 3-3