

Questions on 3.2/3.3 HW?

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With this in mind radicals can be simplified. See the examples below.

<p>Example 1: Simplify $\sqrt{20}$</p> <p>$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{2 \cdot 2 \cdot 5} = 2\sqrt{5}$</p>	<p>Example 2: Simplify $\sqrt[5]{96}$</p> <p>$\sqrt[5]{96} = \sqrt[5]{2^5 \cdot 3} = 2\sqrt[5]{3}$</p>
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Handwritten notes: $\sqrt{2^2 \cdot 5} = 2\sqrt{5}$ (above Example 1), $\sqrt[5]{2^5 \cdot 3} = 2\sqrt[5]{3}$ (above Example 2)

Simplify each of the radicals.

1. $\sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4 \cdot 10} = 2\sqrt{10}$	2. $\sqrt{50}$	3. $\sqrt[3]{16}$
4. $\sqrt{72}$	5. $\sqrt[4]{81} = 3$	6. $\sqrt{32}$
7. $\sqrt[5]{160}$	8. $\sqrt{45}$	9. $\sqrt[3]{54} = \sqrt[3]{2 \cdot 3 \cdot 9} = \sqrt[3]{2 \cdot 3 \cdot 3 \cdot 3} = \sqrt[3]{2} \cdot \sqrt[3]{3^3} = 3\sqrt[3]{2}$

Handwritten notes: $\sqrt{4 \cdot 10} = 2\sqrt{10}$ (above 1), $\sqrt[3]{2^5 \cdot 3} = 2\sqrt[3]{3}$ (above 3), $\sqrt[3]{2 \cdot 27} = \sqrt[3]{2 \cdot 3 \cdot 9} = \sqrt[3]{2 \cdot 3 \cdot 3 \cdot 3} = \sqrt[3]{2} \cdot \sqrt[3]{3^3} = 3\sqrt[3]{2}$ (above 9)

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Find the *arithmetic* means for the following. Show your work.

10.

x	1	2	3
y	5		11

11. $18 + 4x = -10$

x	1	2	3	4	5
y	18	a. 11	b. 4	c. -3	-10

12. $\frac{4x}{4} = \frac{-28}{4}$ $\frac{4x}{4} = -7$ $+x$

x	1	2	3	4	5	6	7
y	12	a.	b.	c.	d.	e.	-6

Find the *geometric* means for the following. Show your work.

13.

x	1	2	3
y	3		12

14.

x	1	2	3	4
y	7	a.	b.	875

15.

x	1	2	3	4	5	6
y						

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14.

x	1	2	3	4
y	7	a.	b.	875

15.

x	1	2	3	4	5	6
y	4	a. 12	b. 36	c.	d.	972

$\frac{4x^5}{x} = \frac{972}{4}$

Fill in the tables of values and find the factor used to move between whole number values, F_w , as well as the factor, F_c , used to move between each column of the table.

16.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
y	4	a. 8	16	b.	c. 64

d. $F_w = 4$
e. $F_c = 4^{1/2}$

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Find the desired values for each function below.

<p>19. $f(x) = 2x - 7$</p> <p>a. Find $f(-3) = 2 \cdot -3 - 7 = -13$ $(-3, -13)$</p> <p>b. Find $f(x) = 21$ $21 = 2x - 7$ $+7 \quad +7$ $28 = 2x$ $\frac{28}{2} = \frac{2x}{2}$ $14 = x$ $(14, 21)$</p> <p>c. Find $f(\frac{1}{2}) = 2(\frac{1}{2}) - 7 = 1 - 7 = -6$ $(\frac{1}{2}, -6)$</p>	<p>20. $g(x) = 3^x(2)$</p> <p>a. Find $g(-4)$</p> <p>b. Find $g(x) = 162$</p> <p>c. Find $g(\frac{1}{2})$</p>	<p>21. $I(t) = 210(1.08^t)$</p> <p>a. Find $I(12)$</p> <p>b. Find $I(t) = 420$</p> <p>c. Find $I(\frac{1}{2})$</p>
<p>22. $h(x) = x^2 + x - 6$</p> <p>a. Find $h(-5)$</p> <p>b. Find $h(x) = 0$</p> <p>c. Find $h(\frac{1}{2})$</p>	<p>23. $k(x) = -5x + 9$</p> <p>a. Find $k(-7)$</p> <p>b. Find $k(x) = 0$</p> <p>c. Find $k(\frac{1}{2})$</p>	<p>24. $m(x) = (5^x)2$</p> <p>a. Find $m(-2)$</p> <p>b. Find $m(x) = 1$</p> <p>c. Find $m(\frac{1}{2})$</p>

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Provide at least three other equivalent forms of the exponential expression. Use rules of exponents such as $3^5 \cdot 3^6 = 3^{11}$ and $(5^2)^3 = 5^6$ as well as division properties and others.

	1 st Equivalent Form	2 nd Equivalent Form	3 rd Equivalent Form
7. $2^{10} = \frac{1}{2^{-10}}$	$(2^5)^2$ or $(2^2)^5$	$2^2 \cdot 2^8$ or $2^5 \cdot 2^5$	$\frac{2^{12}}{2^2}$ or $\frac{2^{20}}{2^{10}}$
8. $3^7 =$			
9. $13^{-8} =$			
10. $7^{\frac{1}{3}} =$			
11. $5^1 =$			

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16. $30(1.05^x)$ $30(1.05^{\frac{1}{7}})^{7x}$ $30(1.05^{\frac{x}{2}})^2$
equivalent $3(1.05^{\frac{1}{7} \cdot 7x}) = 30(1.05^x)$ $30(1.05^{\frac{x}{2} \cdot 2})$

17. $20(1.1^x)$ $20(1.1^{-1})^{-1x}$ $20(1.1^{\frac{1}{5}})^{5x} = 30(1.05^x)$

Go
 Topic: Using rules of exponents

Simplify each expression. Your answer should still be in exponential form.

18. $7^3 \cdot 7^5 \cdot 7^2$	19. $(3^4)^5$	20. $(5^3)^4 \cdot 5^7$
21. $x^3 \cdot x^5$	22. x^{-b}	23. $x^a \cdot x^b$
24. $(x^a)^b$	25. $\frac{y^a}{y^b}$	26. $\frac{(y^a)^c}{y^b}$
27. $(3^4)^6$	28. $\frac{r^5 s^3}{-}$	29. $\frac{x^5 y^{12} z^0}{-}$

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Topic: Using rules of exponents

Simplify each expression. Your answer should still be in exponential form.

18. $7^3 \cdot 7^5 \cdot 7^2$
 $7^{3+5+2} = 7^{10}$

19. $(3^4)^5$

20. $(5^3)^4 \cdot 5^7$

21. $x^3 \cdot x^5$

22. x^{-b}

23. $x^a \cdot x^b$

24. $(x^a)^b = x^{a \cdot b} = x^{ab}$

25. $\frac{y^a}{y^b}$

26. $\frac{(y^a)^c}{y^b}$

27. $\frac{(3^4)^6}{3^7}$

28. $\frac{r^5 s^3}{r s^2}$

29. $\frac{x^5 y^{12} z^0}{x^8 y^9}$
 $= x^{5-8} y^{12-9} = x^{-3} y^3 = \frac{y^3}{x^3}$

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3.3 More Interesting!

A Solidify Understanding Task

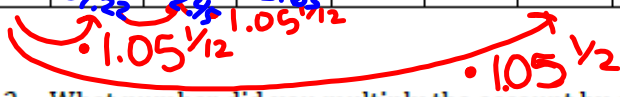


Carlos now knows he can calculate the amount of interest earned on an account in smaller increments than one full year. He would like to determine how much money is in an account each month that earns 5% annually with an initial deposit of \$300.

He starts by considering the amount in the account each month during the first year. He knows that by the end of the year the account balance should be \$315, since it increases 5% during the year.

1. Complete the table showing what amount is in the account each month during the first twelve months.

deposit	1m	2m	3m	4m	5m	6m	7m	8m	9m	10m	11m	12m
\$300	301.25	302.51	303.63									\$315



2. What number did you multiply the account by each month to get the next month's balance?

$1.05^{1/12}$

Carlos knows the exponential equation that gives the account balance for this account on an annual basis is $A = 300(1.05)^t$

Based on his work finding the account balance each month, Carlos writes the following equation for the same account: $A = 300(1.05^{1/12})^{12t}$

3. Verify that both equations give the same results. Using the properties of exponents, explain why these two equations are equivalent.
4. What is the meaning of the $12t$ in this equation?

The 12 months in a year

Carlos shows his equation to Clarita. She suggests his equation could also be approximated by $A = 300(1.004)^{12t}$, since $(1.05)^{1/12} \approx 1.004$. Carlos replies, "I know the 1.05 in the equation $A = 300(1.05)^t$ means I am earning 5% interest annually, but what does the 1.004 mean in this equation?"

5. Answer Carlos' question. What does the 1.004 mean in $A = 300(1.004)^{12t}$?

Interest added on every month

The properties of exponents can be used to explain why $[(1.05)^{1/12}]^{12t} = 1.05^t$. Here are some more examples of using the properties of exponents with rational exponents. For each of the following, simplify the expression using the properties of exponents, and explain what the expression means in terms of the context.

6. $(1.05)^{1/12} \cdot (1.05)^{1/12} \cdot (1.05)^{1/12}$

7. $[(1.05)^{1/12}]^6$

8. $(1.05)^{-1/12}$

9. $(1.05)^2 \cdot (1.05)^{1/4}$

10. $\frac{(1.05)^2}{(1.05)^{1/2}}$

Carlos and Clarita now have two equations representing the balance of their 5% account after t years: $A = 300(1.05)^t$ and $A = 300(1.05^{\frac{1}{12}})^{12t}$. In both of these equations t represents the amount of time the money has been in the account in terms of *years*.

Carlos and Clarita know they can use their equations for fractions of a year by expressing t in terms of a portion of a year, for example, using $t = 2.5$ for two and one-half years or $t = \frac{1}{12}$ for one month. They are wondering if they can write an equation that would find the account balance in terms of t months or t days.

11. Write an equation that will find the account balance in terms of t months.

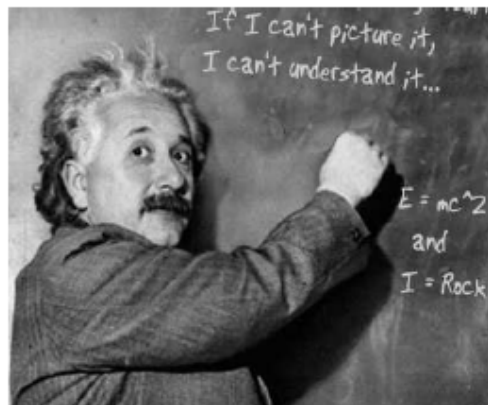
12. Write an equation that will find the account balance in terms of t days.

13. The account balance is currently \$382.88. Write an equation that will find the account balance t months ago.

3.4 Radical Ideas

A Practice Understanding Task

Now that Tia and Tehani know that $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ they are wondering which form, radical form or exponential form, is best to use when working with numerical and algebraic expressions.



Tia says she prefers radicals since she understands the following properties for radicals (and there are not too many properties to remember):

If n is a positive integer greater than 1 and both a and b are positive real numbers then,

$$1. \sqrt[n]{a^n} = a$$

$$\sqrt{2^2} = 2 \quad \sqrt[3]{7^3} = 7$$

$$2. \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt[3]{54} = \sqrt[3]{27} \cdot \sqrt[3]{2}$$

$$3. \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}}$$

Tehania says she prefers exponents since she understands the following properties for exponents (and there are more properties to work with):

$$1. a^m \cdot a^n = a^{m+n}$$

$$2. (a^m)^n = a^{mn}$$

$$3. (ab)^n = a^n \cdot b^n$$

$$4. \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$5. \frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$$

$$6. a^{-n} = \frac{1}{a^n}$$

DO THIS: Illustrate with examples and explain, using the properties of radicals and exponents, why

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$ and $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ are true identities.

Using their preferred notation, Tia might simplify $\sqrt[3]{x^8}$ as follows:

$$\sqrt[3]{x^8} = \sqrt[3]{x^3 \cdot x^3 \cdot x^2} = \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^2} = x \cdot x \cdot \sqrt[3]{x^2} = x^2 \cdot \sqrt[3]{x^2}$$

(Tehani points out that Tia also used some exponent rules in her work.)

On the other hand, Tehani might simplify $\sqrt[3]{x^8}$ as follows:

$$\sqrt[3]{x^8} = x^{8/3} = x^{2+2/3} = x^2 \cdot x^{2/3} \text{ or } x^2 \cdot \sqrt[3]{x^2}$$

For each of the following problems, simplify the expression in the ways you think Tia and Tehani might do it.

Original expression	What Tia and Tehani might do to simplify the expression:
$\sqrt[3]{27}$	Tia's method $\sqrt[3]{9 \cdot 3} = \sqrt[3]{9} \cdot \sqrt[3]{3} = 3\sqrt[3]{3}$
	Tehani's method $\sqrt[3]{3^3} = 3^{3/2} = 3^{1+1/2} = 3^1 \cdot 3^{1/2} = 3\sqrt[3]{3}$
$\sqrt[3]{32}$	Tia's method $\sqrt[3]{2^5} = \sqrt[3]{2^3} \cdot \sqrt[3]{2^2} = 2\sqrt[3]{2^2}$
	Tehani's method
$\sqrt{20x^7}$	Tia's method
	Tehani's method
$\sqrt[3]{\frac{16xy^5}{x^7y^2}}$	Tia's method
	Tehani's method

Tia and Tehani continue to use their preferred notation when solving equations.

For example, Tia might solve the equation $(x + 4)^3 = 27$ as follows:

$$(x + 4)^3 = 27$$

$$\sqrt[3]{(x + 4)^3} = \sqrt[3]{27} = \sqrt[3]{3^3}$$

$$x + 4 = 3$$

$$x = -1$$

Tehani might solve the same equation as follows:

$$(x + 4)^3 = 27$$

$$[(x + 4)^3]^{1/3} = 27^{1/3} = (3^3)^{1/3}$$

$$x + 4 = 3$$

$$x = -1$$

For each of the following problems, simplify the expression in the ways you think Tia and Tehani might do it.

Original equation	What Tia and Tehani might do to solve the equation:
$(x - 2)^2 = 50$	Tia's method
	Tehani's method

$9(x - 3)^2 = 4$	Tia's method
	Tehani's method

Zac is showing off his new graphing calculator to Tia and Tehani. He is particularly excited about how his calculator will help him visualize the solutions to equations.

"Look," Zac says. "I treat the equation like a system of two equations. I set the expression on the left equal to y_1 and the expression on the right equal to y_2 , and I know at the x value where the graphs intersect the expressions are equal to each other."

Zac shows off his new method on both of the equations Tia and Tehani solved using the properties of radicals and exponents. To everyone's surprise, both equations have a second solution.

1. Use Zac's graphical method to show that both of these equations have two solutions. Determine the exact values of the solutions you find on the calculator that Tia and Tehani did not find using their algebraic methods.

Tia and Tehani are surprised when they realize that both of these equations have more than one answer.

2. Explain why there is a second solution to each of these problems.
3. Modify Tia's and Tehani's algebraic approaches so they will find both solutions.
4. Use Zac's graphing calculator approach to solve the following problem.

Carlos and Clarita deposited \$300 in an account earning 5% interest. They want to take the money out of the account when it has doubled in value. To the nearest month, when can they withdraw their money?

Homework

Finish 3.4 "Ready, Set, Go"