Questions on Worksheet 3-1?

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

10. The graph of a function, $f(x)$ is represented at the right. What is the value of $\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$?

$x=3$

$f'(3)$

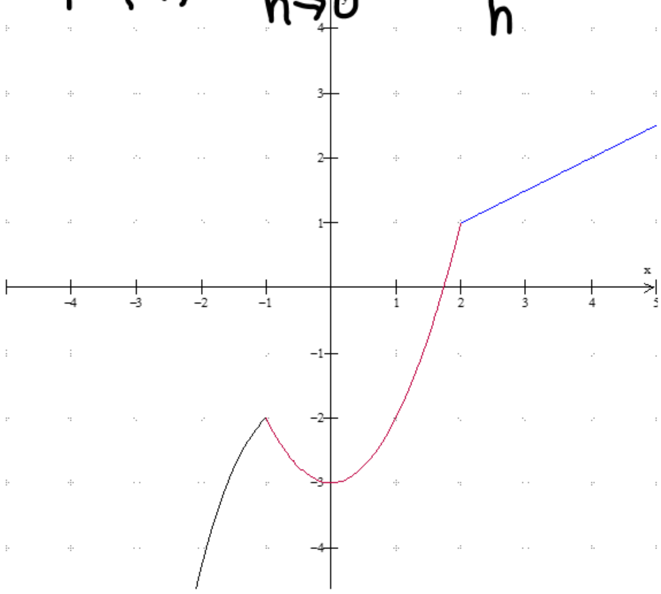
(A) 0

(B) ½

(C) 1

(D) 1 ½

(E) DNE



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9. $f(x) = x + \frac{1}{x}$; $(1, 2)$

$$f'(x) = \lim_{h\to 0} \frac{\frac{(x+h)}{(x+h)}(x+h) + \frac{1}{x+h} - \frac{x}{x}\left(x + \frac{1}{x}\right)}{h} =$$

$$\lim_{h\to 0} \frac{\frac{x}{x}\cdot\frac{(x+h)^2+1}{x+h} - \frac{(x^2+1)}{x}\cdot\frac{(x+h)}{(x+h)}}{h}$$

$$\lim_{h\to 0} \frac{\frac{x\left[(x+h)^2+1\right] - (x^2+1)(x+h)}{x(x+h)}}{\frac{h}{1}} =$$

$$\lim_{h\to 0} \frac{x(x^2+2xh+h^2+1) - (x^3+x^2h+x+h)}{x(x+h)}\cdot\frac{1}{h}$$

$$\lim_{h\to 0} \frac{\cancel{x^3} + 2x^2h + xh^2 + \cancel{x} - \cancel{x^3} - x^2h - \cancel{x} - h}{xh(x+h)}$$

$$\lim_{h\to 0} \frac{x^2h + xh^2 - h}{xh(x+h)} = \lim_{h\to 0} \frac{\cancel{h}(x^2+xh-1)}{x\cancel{h}(x+h)} =$$

$$\frac{x^2 + x\cdot 0 - 1}{x(x+0)} = \boxed{\frac{x^2-1}{x^2}}\ f'(x)\quad @(1,2)$$

$$f'(1) = \frac{1^2-1}{1^2} = \frac{1-0}{1} = 0$$

tangent line: $y - 2 = 0(x-1)$ $\quad{+2}\quad{+2}$

$y = 2$

4) $\frac{1}{2\sqrt{x-4}}$

6) $-\frac{1}{(x-1)^2}$

(6) $$f'(x) = \lim_{h\to 0} \frac{\frac{(x-1)}{(x-1)}\cdot\frac{1}{(x+h-1)} - \frac{1}{(x-1)}\cdot\frac{(x+h-1)}{(x+h-1)}}{h} =$$

$$\lim_{h\to 0} \frac{\frac{\cancel{x}-\cancel{1}+(\cancel{x}+h-\cancel{1})}{(x-1)(x+h-1)}}{h} = \lim_{h\to 0} \frac{\frac{-h}{(x-1)(x+h-1)}}{\frac{h}{1}}\cdot\frac{1}{h}$$

$$\lim_{h\to 0} \frac{-\cancel{h}}{(x-1)(x+h-1)\cancel{h}} = \lim_{h\to 0} \frac{-1}{(x-1)(x+h-1)} = \frac{-1}{(x-1)(x+0-1)}$$

$$= \frac{-1}{(x-1)(x-1)} = \boxed{\frac{-1}{(x-1)^2}}\ f'(x)$$

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3.2 Differentiability

A function will not have a derivative at a point $P(a, f(a))$

where the slopes of the secant lines, $\dfrac{f(x)-f(a)}{x-a}$

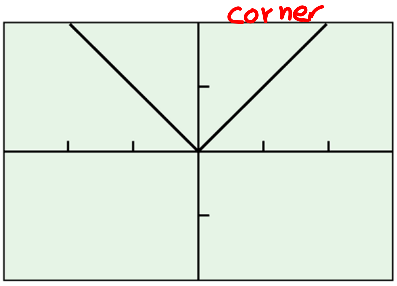
fail to approach a limit as x approaches a.

The next figures illustrate four different instances where this occurs. For example, a function whose graph is otherwise smooth will fail to have a derivative at a point where the graph has:

1. a corner, where the one-sided derivatives differ;

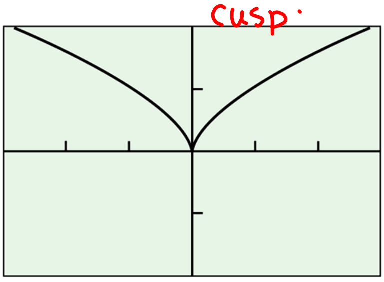
$f(x)=|x|$

corner



[–3, 3] by [–2, 2]

cusp.



[–3, 3] by [–2, 2]

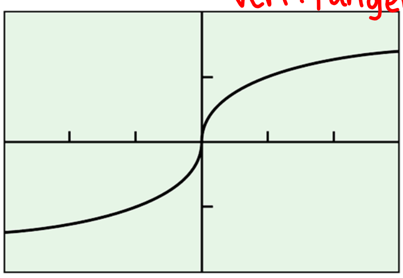
2. a cusp, where the slopes of the secant lines approach ∞ from one side and approach $-\infty$ from the other (an extreme case of a corner);

$f(x)=x^{\frac{2}{3}}$

3. A vertical tangent, where the slopes of the secant lines approach either ∞ or $-\infty$ from both sides;

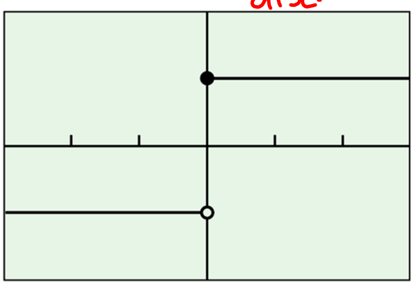
$f(x)=\sqrt[3]{x}$

vert. tangent



[–3, 3] by [–2, 2]

disc.



[–3, 3] by [–2, 2]

4. a discontinuity (which will cause one or both of the one-sided derivatives to be nonexistent).

$$U(x)=\begin{cases}-1, & x<0\\ 1, & x\geq 0\end{cases}$$

Example:

Show that the function is not differentiable at $x=0$.

left

$$f(x)=\begin{cases} x^3, & x\le 0 \\ 4x, & x>0 \end{cases}$$

$(x^2+2xh+h^2)(x+h)=$

$x^3+2x^2h+xh^2$

$x^2h+2xh^2+h^3$

$x^3+3x^2h+3xh^2+h^3$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}=\frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h}=$$

$$\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h}=\lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}=$$

$$3x^2+3x(0)+0^2=\underline{\underline{3x^2}}$$

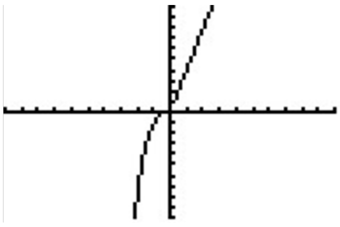
$$f'(0)=3\cdot 0^2=0$$

Right-handed limit:

$$f'(x)=\lim_{h\to 0}\frac{4(x+h)-4x}{h}=\lim_{h\to 0}\frac{4x+4h-4x}{h}$$

$$=\lim_{h\to 0}\frac{4\cancel{h}}{\cancel{h}}=\underline{\underline{4}}$$

The right-hand derivative is 4.
The left-hand derivative is 0.
The function is not differentiable at $x=0$.

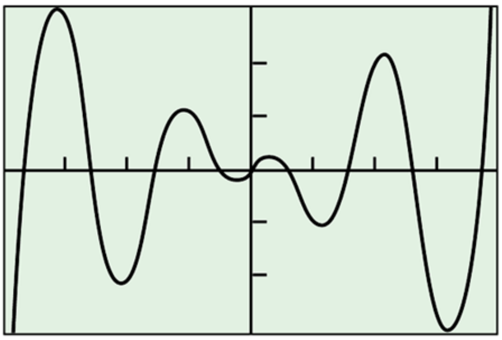


Most of the functions we encounter in calculus are differentiable wherever they are defined, which means they will *not* have corners, cusps, vertical tangent lines or points of discontinuity within their domains. Their graphs will be unbroken and smooth, with a well-defined slope at each point.

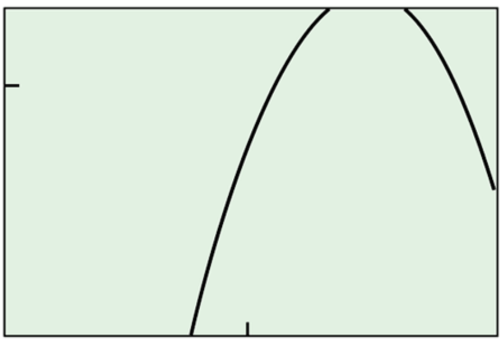
Differentiability Implies Local Linearity

A good way to think of differentiable functions is that they are **locally linear**; that is, a function that is differentiable at a closely resembles its own tangent line very close to a.

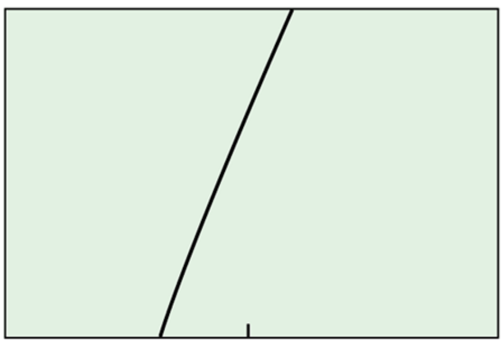
In the jargon of graphing calculators, differentiable curves will "straighten out" when we zoom in on them at a point of differentiability.



[–4, 4] by [–3, 3]
(a)



[1.7, 2.3] by [1.7, 2.1]
(b)



[1.93, 2.07] by [1.85, 1.95]
(c)

Derivatives on a Calculator:

Many graphing utilities can approximate derivatives numerically with good accuracy at most points of their domains. For small values of h, the difference quotient

$$\frac{f(a+h)-f(a)}{h}$$

is often a good numerical approximation of $f'(a)$.
However, the same value of h will usually yield a better approximation if we use the symmetric difference quotient

$$\frac{f(a+h)-f(a-h)}{2h} \rightarrow \frac{f(a+0.001)-f(a-0.001)}{2(0.001)} \rightarrow$$

which is what our graphing calculator uses to calculate NDER $f(a)$, the **numerical derivative of f at a point** a.

$$f(2.001)-f(1.999)$$

The **numerical derivative of f** as a function is denoted by NDER $f(x)$.
The numerical derivatives we compute in this book will use $h=0.001$.

Example:

Find the numerical derivative of the function $f(x)=x^2+3$ at the point $x=2$. Use a calculator with $h=0.001$.

function, x, x-value

Using a TI-83 Plus we get

```
nDeriv(X^2+3,X,2
                4
```

Because of the method used internally by the calculator, you will sometimes get a derivative value at a nondifferentiable point.

This is a case of where you must be "smarter" than the calculator.

Theorem 1: Differentiability Implies Continuity

If f has a derivative at $x=a$, then f is continuous at $x=a$.

The converse of Theorem 1 is false. A continuous functions might have a corner, a cusp or a vertical tangent line, and hence not be differentiable at a given point.

Intermediate Value Theorem for Derivatives

Not every function can be a derivative.

If a and b are any two points in an interval on which f is differentiable, then f' takes on every value between $f'(a)$ and $f'(b)$.

Homework

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