Please get out your Unit 2 HW and pink sheet for me to check off your homework.

3.1 The Derivative

The **derivative** of the function f with respect to the variable x is the function $f'(x)$ whose value at x is

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

provided the limit exists.

The domain of f', the set of points in the domain of f for which the limit exists, may be smaller than the domain of f. If $f'(x)$ exists, we say that f **has a derivative (is differentiable)** at x. A function that is differentiable at every point in its domain is a **differentiable function**.

Example:

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Differentiate f(x) = x^2

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}=\lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}=$$

$$\lim_{h\to 0}\frac{2xh+h^2}{h}=\lim_{h\to 0}\frac{h(2x+h)}{h}=\lim_{h\to 0}2x+h=$$

$$2x+0=\boxed{2x}$$

$f(x)=x^2$

$f'(x)=2x$

Differentiate $f(x)=x^2$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{(x+h)^2-x^2}{h} \quad \text{substitute}$$

$$=\lim_{h\to 0}\frac{(x^2+2xh+h^2)-x^2}{h} \quad (x+h)^2 \text{ expanded}$$

$$=\lim_{h\to 0}\frac{(2x+h)h}{h} \; x^2 \quad \text{cancelled and h factored out}$$

$$=\lim_{h\to 0}(2x+h)$$

$$=2x$$

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Derivative at a Point:

The derivative of the function f at the point where $x = a$ is the limit

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided the limit exists.

Notation:

There are many ways to denote the derivative of a function $y = f(x)$.

Besides $f'(x)$, the most common notations are:

y'	"y prime"	Nice and brief, but does not name the independent variable.
$\frac{dy}{dx}$	"dy dx" or "the derivative of y with respect to x"	Names both variables and uses d for derivative.
$\frac{df}{dx}$	"df dx" or "the derivative of f with respect to x"	Emphasizes the function's name.
$\frac{d}{dx}f(x)$	"d dx of f at x" or "the derivative of f at x"	Emphasis that differentiation is an operation performed on f.

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One-sided Derivatives:

A function $y=f(x)$ is **differentiable on a closed interval** $[\mathbf{a},\mathbf{b}]$ if it has a derivative at every interior point on the interval, and if the limits

$$\lim_{h\to 0^+}\frac{f(a+h)-f(a)}{h} \qquad [\text{the } \textbf{right-hand derivative at a}]$$

$$\lim_{h\to 0^-}\frac{f(b+h)-f(b)}{h} \qquad [\text{the } \textbf{left-hand derivative at b}]$$

exist at the endpoints. In the right-hand derivative, h is positive and $a+h$ approaches a from the right. In the left-hand derivative, h is negative and $b+h$ approaches b from the left.

Right-hand and left-hand derivatives may be defined at any point of a function's domain.

The usual relationship between one-sided and two-sided limits holds for derivatives. Theorem 3, Section 2.1, allows us to conclude that a function has a (two-sided) derivative at a point if and only if the function's right-hand and left-hand derivatives are defined and equal at that point.

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$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Examples:

Find the derivative of

a. $f(x) = \sqrt{2x}$ b. $f(x) = \frac{1}{x^2}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2x+2h} - \sqrt{2x}}{h} \cdot \left(\frac{\sqrt{2x+2h} + \sqrt{2x}}{\sqrt{2x+2h} + \sqrt{2x}}\right) =$$

$$\lim_{h \to 0} \frac{2x + 2h - 2x}{h(\sqrt{2x+2h} + \sqrt{2x})} =$$

$$\lim_{h \to 0} \frac{2h}{h(\sqrt{2x+2h} + \sqrt{2x})}$$

$$\lim_{h \to 0} \frac{2}{(\sqrt{2x+2h} + \sqrt{2x})} =$$

$$\frac{2}{\sqrt{2x+2\cdot 0} + \sqrt{2x}} = \frac{2}{\sqrt{2x} + \sqrt{2x}} =$$

$$\frac{2}{2\sqrt{2x}} = \boxed{\frac{1}{\sqrt{2x}}} \text{ or } (2x)^{-1/2}$$

$(1, 3)$

$$\frac{1}{\sqrt{2 \cdot 1}} = \frac{1}{\sqrt{2}}$$:)

(b) $f(x) = \frac{1}{x^2}$, $f'(x) = \frac{-2}{x^3}$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} \cdot \frac{x^2}{x^2} - \frac{1}{x^2} \cdot \frac{(x+h)^2}{(x+h)^2}}{h} = \lim_{h \to 0} \frac{\frac{x^2 - (x+h)^2}{x^2(x+h)^2}}{h} =$$

$$\lim_{h \to 0} \frac{x^2 - (x^2 + 2xh + h^2)}{x^2 h (x+h)^2} = \lim_{h \to 0} \frac{x^2 - x^2 - 2xh - h^2}{x^2 h (x+h)^2} =$$

$$\lim_{h \to 0} \frac{h(-2x - h)}{x^2 h (x+h)^2} = \lim_{h \to 0} \frac{(-2x - h)}{x^2 (x+h)^2} = \frac{-2x - 0}{x^2 (x+0)^2} = \frac{-2x}{x^2 \cdot x^2}$$

$$= \frac{-2x}{x^4} = \boxed{\frac{-2}{x^3}}$$

Find the derivative of $f(x) = \frac{1}{x}$.

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Homework

Worksheet 3-1