3.9 My Irrational and Imaginary Friends

A Solidify Understanding Task

Questions on 3.9 HW? Finish up page 58.

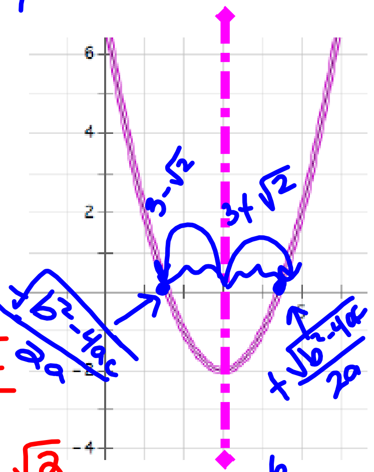
discriminant

The quadratic formula is usually written in the form $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. An equivalent form is $\frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$. If a, b and c are rational coefficients, then $\frac{-b}{2a}$ is a rational term, and $\frac{\sqrt{b^2-4ac}}{2a}$ may be a rational term, an irrational term or an imaginary term, depending on the value of the expression under the square root sign.

axis of symmetry

$a=1 \quad b=-6 \quad c=7$

12. Examine the roots of the quadratic $y = x^2 - 6x + 7$ shown in the graph at the right. How do the terms $\frac{-b}{2a}$ and $\frac{\sqrt{b^2-4ac}}{2a}$ show up in this graph?



$x = \frac{-b}{2a} = \frac{-(-6)}{2 \cdot 1} = \frac{6}{2} = 3$

$\frac{\sqrt{(-6)^2 - 4 \cdot 1 \cdot 7}}{2 \cdot 1} = \frac{\sqrt{36-28}}{2} = \frac{\sqrt{8}}{2} = \frac{\sqrt{\cancel{4}^{2}} \cdot \sqrt{2}}{2} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \sqrt{2}$

Look back at the work you did in the task *To Be Determined*. . .

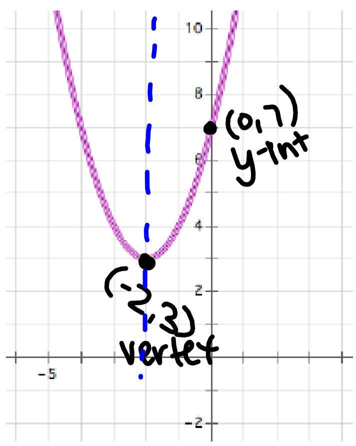
13. Which quadratics in that task had complex roots?

$i = \sqrt{-1}$

$i^2 = -1$

14. How can you determine if a quadratic has complex roots from its graph?

15. Find the complex roots of the following quadratic function represented by its graph.



$+7$

$(x - -2)^2 + 3$

$(x+2)^2 + 3 = 0$

$\quad -3 \quad -3$

$\sqrt{(x+2)^2} = \sqrt{-3}$

$x+2 = \pm\sqrt{-1} \cdot \sqrt{3}$

$x+2 = \pm i\sqrt{3}$

$\quad -2 \quad -2$

$\boxed{x = -2 \pm i\sqrt{3}}$

Note: Complex numbers are not real numbers—they do not lie on the real number line that includes all of the rational and irrational numbers; also note that the real numbers are a subset of the complex numbers since a real number results when the imaginary part of $a + bi$ is 0, that is, $a + 0i$.

Factored Form: $(x - -2 + i\sqrt{3})(x - -2 - i\sqrt{3})$

$(x+2+i\sqrt{3})(x+2-i\sqrt{3})$

Classify each of the numbers represented below according to the sets to which they belong. a number fits in more than one set then list all that apply.
(Whole numbers "𝕎", Integers "ℤ", Rational "ℚ", Irrational "$\overline{\mathbb{Q}}$", Real "ℝ", Complex "ℂ")

1. π
2. -13
3. $\sqrt{-16}$ $\mathbb{C}$
4. 0
5. $\sqrt{75}$
6. $\frac{9}{3}$
7. $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \pm\frac{2}{3}$ $\mathbb{Q}, \mathbb{R}$
8. $5 + \sqrt{2}$ $\overline{\mathbb{Q}}$
9. $\sqrt{-40}$

Set

Topic: Simplifying radicals, imaginary numbers

Simplify each radical expression below.

10. $3 + \sqrt{2} - 7 + 3\sqrt{2}$
11. $\sqrt{5} - 9 + 8\sqrt{5} + 11 - \sqrt{5}$
12. $\sqrt{12} + \sqrt{48}$
13. $\sqrt{8} - \sqrt{18} + \sqrt{32}$
14. $11\sqrt{7} - 5\sqrt{7}$
15. $7\sqrt{7} + 5\sqrt{3} - 3\sqrt{7} + \sqrt{3}$
$4\sqrt{7} + 6\sqrt{3}$

8.50 x 11.00 in

Real # — or — Complex

Rational — Irrational

Rational
Integers
Whole
Natural

SM2-Module 2 SE.pdf - Adobe Acrobat Reader DC

File Edit View Window Help

Home Tools SM2-Module 2 SE.... AP Test.pdf

60 / 69 150%

$i = \sqrt{-1}$

$i^2 = -1$

Simplify. Express as a complex number using " *i* " if necessary.

16. $\sqrt{-2} \cdot \sqrt{-2}$

17. $7 + \sqrt{-25}$

18. $(4i)^2$ $= (4i)(4i) = 16i^2 = 16(-1) = -16$

19. $i^2 \cdot i^3 \cdot i^4$

20. $\left(\sqrt{-4}\right)^3$

21. $(2i)(5i)^2$

8.50 x 11.00 in

SM2-Module 2 SE.pdf - Adobe Acrobat Reader DC

File Edit View Window Help

Home Tools SM2-Module 2 SE.... AP Test.pdf

61 / 69 150%

Solve each quadratic equation over the set of complex numbers.

22. $x^2 + 100 = 0$

23. $t^2 + 24 = 0$

$-24 \quad -24$

$\sqrt{t^2} = \sqrt{-24}$

$t = \pm\sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{6}$

$i \quad 2$

$\boxed{t = \pm 2i\sqrt{6}}$

24. $x^2 - 6x + 13 = 0$

25. $r^2 - 2r + 5 = 0$

Go

Topic: Solve quadratic equations.

Use the discriminant to determine the nature of the roots to the quadratic equation.

$b^2 - 4ac$

26. $x^2 - 5x + 7 = 0$

$a=1 \quad c=7$

$b=5$

$5^2 - 4\cdot1\cdot7$

$25 - 28$

-3

complex roots

27. $x^2 - 5x + 6 = 0$

28. $2x^2 - 5x + 5 = 0$

29. $x^2 + 7x + 2 = 0$

30. $2x^2 + 7x + 6 = 0$

31. $2x^2 + 7x + 7 = 0$

8.50 x 11.00 in

32. $2x^2 - 7x + 6 = 0$

33. $2x^2 + 7x - 6 = 0$

34. $x^2 + 6x + 9 = 0$

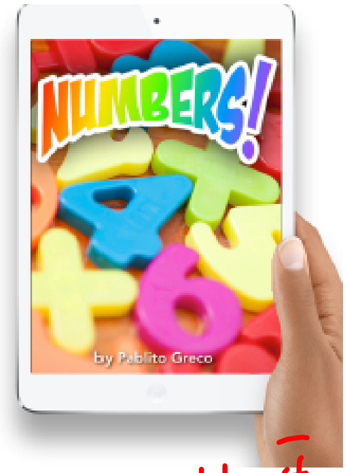
Solve the quadratic equations below using an appropriate method.

35. $m^2 + 15m + 56 = 0$

36. $5x^2 - 3x + 7 = 0$

37. $x^2 - 10x + 21 = 0$

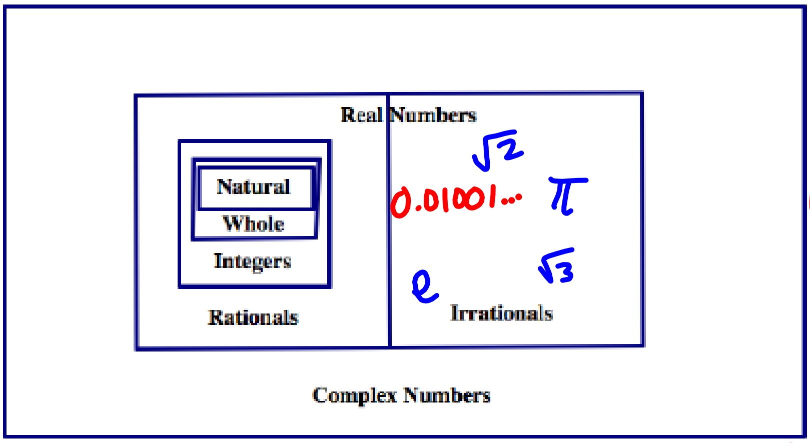
38. $6x^2 + 7x - 5 = 0$



3.10 iNumbers

A Practice Understanding Task

In order to find solutions to all quadratic equations, we have had to extend the number system to include complex numbers.



difference - subtract −
quotient - divide ÷
product - multiply ·
sum - add +

Do the following for each of the problems below:

- Choose the best word to complete each conjecture.
- After you have made a conjecture, create at least four examples to show why your conjecture is true.
- If you find a counter-example, change your conjecture to fit your work.

Conjecture #1: The sum of two integers is [always, sometime, never] an integer.

$-2+2=0$ $\quad 3+-7=-4$

$-3+7=4$ $\quad 8+0=8$

Conjecture #2: The sum of two rational numbers is [always, sometimes, never] a rational number.

Conjecture #3: The sum of two irrational numbers is [always, sometimes, never] an irrational number.

Conjecture #4: The sum of two real numbers is [always, sometimes, never] a real number.

Conjecture #5: The sum of two complex numbers is [always, sometimes, never] a complex number.

Conjecture #6: The product of two integers is [always, sometime, never] an integer.

Conjecture #7: The quotient of two integers is [always, sometime, never] an integer.

Conjecture #8: The product of two rational numbers is [always, sometimes, never] a rational number.

Conjecture #9: The quotient of two rational numbers is [always, sometimes, never] a rational number

Conjecture #10: The product of two irrational numbers is [always, sometimes, never] an irrational number.

Conjecture #11: The product of two real numbers is [always, sometimes, never] a real number.

Conjecture #12: The product of two complex numbers is [always, sometimes, never] a complex number.

13. The ratio of the circumference of a circle to its diameter is given by the irrational number π. Can the diameter of a circle and the circumference of the same circle both be rational numbers? Explain why or why not.

The Arithmetic of Polynomials

In the task *To Be Determined . . .* we defined polynomials to be expressions of the following form:

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$$

where all of the exponents are positive integers and all of the coefficients $a_0 \ldots a_n$ are constants.

2 $\quad x \quad$ x^2+x+7 $\quad 2x^2-3x+7$

Do the following for each of the problems below:

- Choose the best word to complete each conjecture.
- After you have made a conjecture, create at least four examples to show why your conjecture is true.
- If you find a counter-example, change your conjecture to fit your work.

Conjecture #P1: The sum of two polynomials is [always, sometime, never] a polynomial.

Conjecture #P2: The difference of two polynomials is [always, sometime, never] a polynomial.

Conjecture #P3: The product of two polynomials is [always, sometime, never] a polynomial.

Homework

3.10 MVP "Ready, Set, Go"