3.9 My Irrational and Imaginary Friends

A Solidify Understanding Task

Questions on 3.9 HW? Take ten minutes to finish up ~~page 58.~~

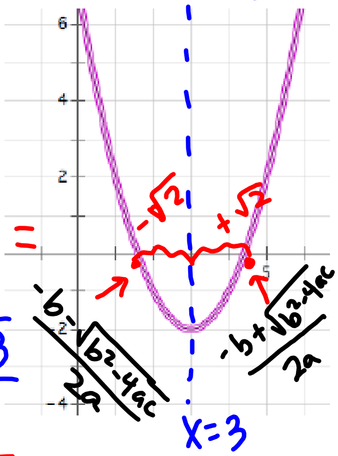
discriminant b^2-4ac

The quadratic formula is usually written in the form $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. An equivalent form is $\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$. If a, b and c are rational coefficients, then $\frac{-b}{2a}$ is a rational term, and $\frac{\sqrt{b^2-4ac}}{2a}$ may be a rational term, an irrational term or an imaginary term, depending on the value of the expression under the square root sign.

axis of symmetry

$a=1 \quad b=-6 \quad c=7$

12. Examine the roots of the quadratic $y=x^2-6x+7$ shown in the graph at the right. How do the terms $\frac{-b}{2a}$ and $\frac{\sqrt{b^2-4ac}}{2a}$ show up in this graph?



$x=-\frac{b}{2a}$

$x=\frac{--6}{2\cdot 1}=\boxed{3}$

$\frac{\sqrt{(-6)^2-4\cdot1\cdot7}}{2\cdot1}=$

$\frac{\sqrt{36-28}}{2}=\frac{\sqrt{8}}{2}$

$=\frac{\cancel{\sqrt{4}}^{\,2}\cdot\sqrt{2}}{2}=\frac{2\sqrt{2}}{2}=\sqrt{2}$

Look back at the work you did in the task *To Be Determined . . .*

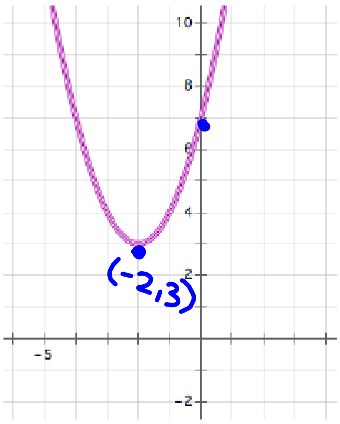
13. Which quadratics in that task had complex roots?

14. How can you determine if a quadratic has complex roots from its graph?

15. Find the complex roots of the following quadratic function represented by its graph.

$i\,\cancel{\sqrt{-1}}\cdot\sqrt{3}$

$\pm i\sqrt{3}$



$\underline{\qquad\qquad}+c$

$(x--2)^2+3$

$(x+2)^2+3=0$
$\quad\ -3 \quad -3$

$\sqrt{(x+2)^2}=\sqrt{-3}$

$x+2=\pm\sqrt{-3}$

$x+2=\pm i\sqrt{-3}$
$\ -2 \quad -2$

$x=-2\pm i\sqrt{-3}$

Note: Complex numbers are not real numbers—they do not lie on the real number line that includes all of the rational and irrational numbers; also note that the real numbers are a subset of the complex numbers since a real number results when the imaginary part of $a+bi$ is 0, that is, $a+0i$.

Factored Form: $(x--2+i\sqrt{3})(x--2-i\sqrt{3})$

$(x+2+i\sqrt{3})(x+2-i\sqrt{3})$

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Classify each of the numbers represented below according to the sets to which they belong. If a number fits in more than one set then list all that apply.
(Whole numbers "$\mathbb{W}$", Integers "$\mathbb{Z}$", Rational "$\mathbb{Q}$", Irrational "$\overline{\mathbb{Q}}$", Real "$\mathbb{R}$", Complex "$\mathbb{C}$")

1. π
2. -13
3. $\sqrt{-16}$ $\mathbb{C}$
4. 0
5. $\sqrt{75} = \sqrt{25}\cdot\sqrt{3} = \pm 5\sqrt{3}$ $\overline{\mathbb{Q}}, \mathbb{R}$
6. $\frac{9}{3}$
7. $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \pm\frac{2}{3}$ $\mathbb{Q}, \mathbb{R}$
8. $5 + \sqrt{2}$ $\overline{\mathbb{Q}}, \mathbb{R}$
9. $\sqrt{-40}$

Set

Topic: Simplifying radicals, imaginary numbers

Simplify each radical expression below.

10. $3 + \sqrt{2} - 7 + 3\sqrt{2}$
11. $\sqrt{5} - 9 + 8\sqrt{5} + 11 - \sqrt{5}$
12. $\sqrt{12} + \sqrt{48}$
13. $\sqrt{8} - \sqrt{18} + \sqrt{32}$
14. $11\sqrt{7} - 5\sqrt{7}$
15. $7\sqrt{7} + 5\sqrt{3} - 3\sqrt{7} + \sqrt{3}$

Real —— OR —— Complex
i

Real → Rational, Irrational

fractions Rational
neg & pos Integers
0, pos Whole
1, 2, 3, ... Counting / natural

Irrational
π, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$

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Simplify each radical expression below.

$i = \sqrt{-1}$

$i^2 = -1$

10. $3 + \sqrt{2} - 7 + 3\sqrt{2}$

11. $\sqrt{5} - 9 + 8\sqrt{5} + 11 - \sqrt{5}$

12. $\sqrt{12} + \sqrt{48}$

13. $\sqrt{8} - \sqrt{18} + \sqrt{32}$

14. $11\sqrt{7} - 5\sqrt{7}$

15. $7\sqrt{7} + 5\sqrt{3} - 3\sqrt{7} + \sqrt{3}$

$4\sqrt{7} + 6\sqrt{3}$

Simplify. Express as a complex number using " *i* " if necessary.

16. $\sqrt{-2} \cdot \sqrt{-2}$

$\sqrt{4} = \pm 2$

17. $7 + \sqrt{-25}$

18. $(4i)^2$

19. $i^2 \cdot i^3 \cdot i^4$

20. $(\sqrt{-4})^3$

21. $(2i)(5i)^2$

$2i(25i^2)$

$2i(-25)$

$-50i$

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61 / 69 150%

Topic: Solve quadratic equations.

b^2-4ac

Use the discriminant to determine the nature of the roots to the quadratic equation.

$a=1$
$b=5$
$c=7$

26. $x^2 - 5x + 7 = 0$

$(-5)^2-4\cdot1\cdot7$

$25-28=-3$

complex roots

27. $x^2 - 5x + 6 = 0$

28. $2x^2 - 5x + 5 = 0$

29. $x^2 + 7x + 2 = 0$

30. $2x^2 + 7x + 6 = 0$

31. $2x^2 + 7x + 7 = 0$

32. $2x^2 - 7x + 6 = 0$

33. $2x^2 + 7x - 6 = 0$

34. $x^2 + 6x + 9 = 0$

Solve the quadratic equations below using an appropriate method.

35. $m^2 + 15m + 56 = 0$

36. $5x^2 - 3x + 7 = 0$

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61 / 69 150%

Solve each quadratic equation over the set of complex numbers.

22. $x^2 + 100 = 0$

23. $t^2 + 24 = 0$

$$-24 \quad -24$$

$$\sqrt{t^2} = \sqrt{-24}$$

$$t = \pm \cancel{\sqrt{-1}} \cdot \cancel{\sqrt{4}} \cdot \sqrt{6}$$

$$i \quad 2$$

$$t = \pm 2i\sqrt{6}$$

24. $x^2 - 6x + 13 = 0$

25. $r^2 - 2r + 5 = 0$

Go

Topic: Solve quadratic equations.

Use the discriminant to determine the nature of the roots to the quadratic equation.

26. $x^2 - 5x + 7 = 0$

27. $x^2 - 5x + 6 = 0$

28. $2x^2 - 5x + 5 = 0$

29. $x^2 + 7x + 2 = 0$

30. $2x^2 + 7x + 6 = 0$

31. $2x^2 + 7x + 7 = 0$

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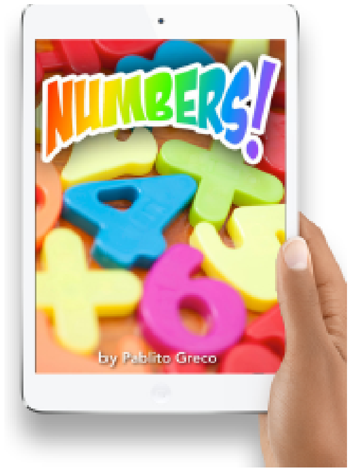
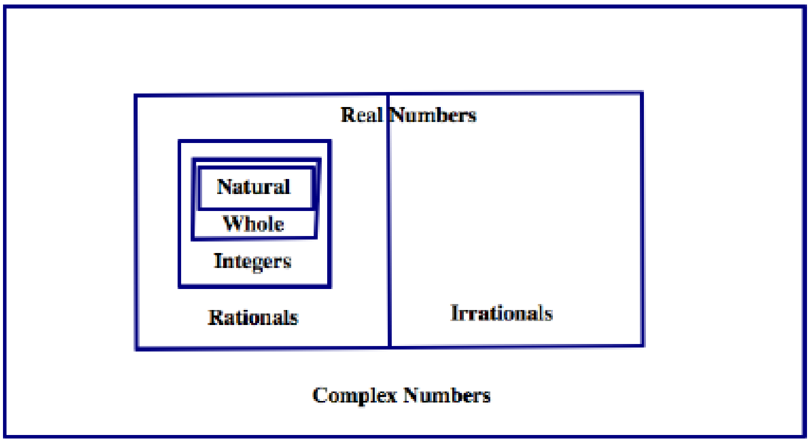


3.10 iNumbers

A Practice Understanding Task

In order to find solutions to all quadratic equations, we have had to extend the number system to include complex numbers.



Do the following for each of the problems below:

- Choose the best word to complete each conjecture.
- After you have made a conjecture, create at least four examples to show why your conjecture is true.
- If you find a counter-example, change your conjecture to fit your work.

Conjecture #1: The sum of two integers is [always, sometime, never] an integer.

$-2+4=2$ $-7+3=-4$ $-3+3=0$

$8+0=8$

Conjecture #2: The sum of two rational numbers is [always, sometimes, never] a rational number.

Conjecture #3: The sum of two irrational numbers is [always, sometimes, never] an irrational number.

Conjecture #4: The sum of two real numbers is [always, sometimes, never] a real number.

Conjecture #5: The sum of two complex numbers is [always, sometimes, never] a complex number.

Conjecture #6: The product of two integers is [always, sometime, never] an integer.

Conjecture #7: The quotient of two integers is [always, sometime, never] an integer.

Conjecture #8: The product of two rational numbers is [always, sometimes, never] a rational number.

Conjecture #9: The quotient of two rational numbers is [always, sometimes, never] a rational number

Conjecture #10: The product of two irrational numbers is [always, sometimes, never] an irrational number.

Conjecture #11: The product of two real numbers is [always, sometimes, never] a real number.

Conjecture #12: The product of two complex numbers is [always, sometimes, never] a complex number.

13. The ratio of the circumference of a circle to its diameter is given by the irrational number π. Can the diameter of a circle and the circumference of the same circle both be rational numbers? Explain why or why not.

The Arithmetic of Polynomials

In the task *To Be Determined . . .* we defined polynomials to be expressions of the following form:

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$$

where all of the exponents are positive integers and all of the coefficients $a_0 \ldots a_n$ are constants.

x^2

$x^2 - 3x + 4$

4

Do the following for each of the problems below:

- Choose the best word to complete each conjecture.
- After you have made a conjecture, create at least four examples to show why your conjecture is true.
- If you find a counter-example, change your conjecture to fit your work.

Conjecture #P1: The sum of two polynomials is [always, sometime, never] a polynomial.

Conjecture #P2: The difference of two polynomials is [always, sometime, never] a polynomial.

Conjecture #P3: The product of two polynomials is [always, sometime, never] a polynomial.

Homework

3.10 MVP "Ready, Set, Go"

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