

Questions on 2.6 HW? 2.4 HW is due today...and there's a quiz...

Part II

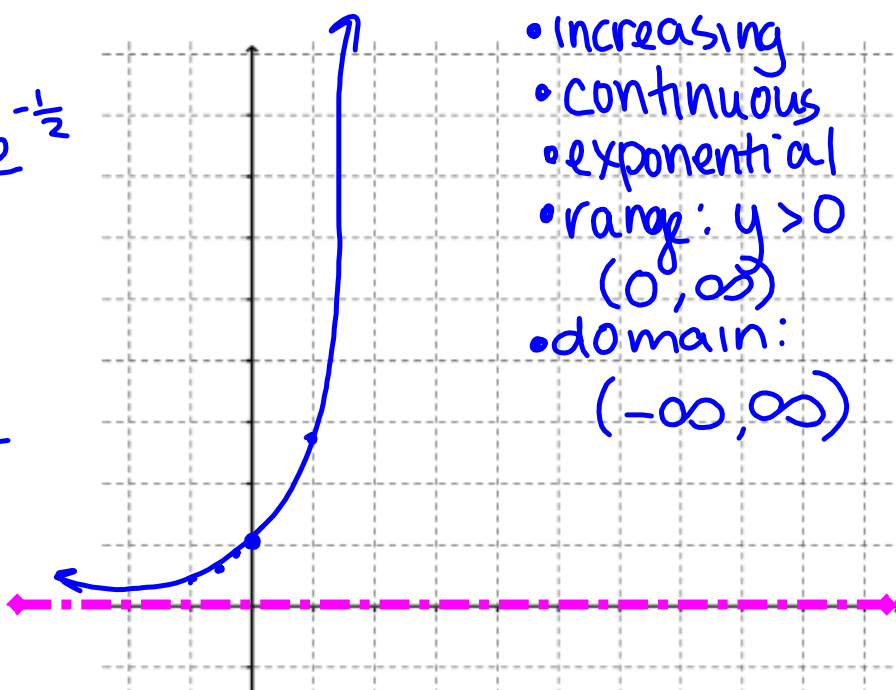
Since e is widely used to model exponential growth and decay in many contexts, let's get a little more familiar with the base e exponential function:

$$f(x) = e^x$$

$$f(x) = Pe^{rt}$$

1. Make a prediction about the graph of $f(x)$. Explain what knowledge you used to make your prediction.
2. Create a table and a graph and describe the mathematical features of $f(x)$.

x	y
$-\frac{1}{2}$	0.6 $e^{-\frac{1}{2}}$
$-\frac{1}{4}$	0.78
0	1
1	2.72
2	7.4
3	20.1



- increasing
- continuous
- exponential
- range: $y > 0$
 $(0, \infty)$
- domain:
 $(-\infty, \infty)$

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1.

x	y
-3	-23
-2	-17
-1	-11
0	-5
1	1
2	7
3	13

a. Pattern:
 b. Recursive equation:

2.

x	y
-3	4
-2	0
-1	-2
0	-2
1	0
2	4
3	10

a. Pattern:
 b. Recursive equation:

Quadratic
 x^2

3.

x	y
-3	-15
-2	-10
-1	-5
0	0
1	5
2	10
3	15

a. Pattern:
 b. Recursive equation:

constant
 x^0

4.

x	y
-3	24
-2	22
-1	20
0	18
1	16
2	14
3	12

a. Pattern:
 b. Recursive equation:

5.

x	y
-3	48
-2	22
-1	6
0	0
1	4
2	18
3	42

a. Pattern:
 b. Recursive equation:

6.

x	y
-3	4
-2	1
-1	0
0	1
1	4
2	9
3	16

a. Pattern:
 b. Recursive equation:

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19. $3^{\log_3 81} = \underline{81}$ because $\log_3 81$ equals the exponent 4 that makes $3^? = 81$.

20. $a^{\log_a x} = \underline{x}$ because $\log_a x$ equals the exponent that makes $a^? = x$. ^{? = $\log_a x$}

$g(x) = \log_3 x$
 $f(x) = 3^x$

$f(g(x)) = 3^{\log_3 x}$

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Secondary Mathematics III

8.50 x 11.00 in

Logarithms Quiz #3: Interest

Use the interest formulas to determine how much you will have after 10 years in an account that you place \$200 in with a 2% rate.

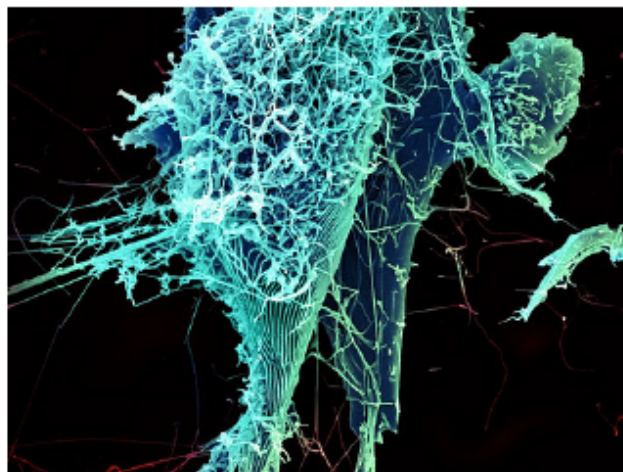
$$A = P\left(1 + \frac{r}{n}\right)^{nt} \quad A = Pe^{rt}$$

- 1) compounded semi-annually
- 2) compounded continuously

2.7H Logs Go Viral

A Solidify Understanding Task

As we learned in *Compounding the Problem*, when money is compounded continuously, it turns out that the base of the exponential growth function is $e \approx 2.71828$. It turns out that basically all situations that grow or decay continuously can be modeled with a base e exponential function. The basic formulas are:



Continuous growth:

$$A = Pe^{rt}$$

Continuous Decay:

$$A = Pe^{-rt}$$

In both cases, A is the amount of "stuff" at time t . P is the amount of "stuff" to start with, or at $t = 0$, and r is the rate of growth or decay. So, if we're talking about money, A and P are dollar amounts. If we're talking about a population, A and P are the number of people in the population. If we're talking about radioactive decay, then A and P are the mass of the radioactive substance. In other words, P is what we started with and A is what we have after it grows or decays.

Since there are many things that grow or decay continuously in nature, e is called the natural exponential function and the base e logarithm is called the natural logarithm. Think about the growth of bacteria. One cell splits into two cells. The two cells begin to split, and the resulting cells split and so on. In a group of many cells, there would be a cell splitting nearly continuously, thus it is a base e exponential function. So, to use this base e exponential and its inverse, imagine the following scenarios:

$$\log_e x = \ln x$$

You are an epidemiologist, a person who studies the outbreak and spread of diseases. Part of your job is to help avoid a pandemic—a worldwide outbreak of a disease. You know that some of the most difficult diseases to deal with are viruses because they don't respond to many of the medicines that we have available and because viruses are able to mutate and change quickly, making it more difficult to contain them.

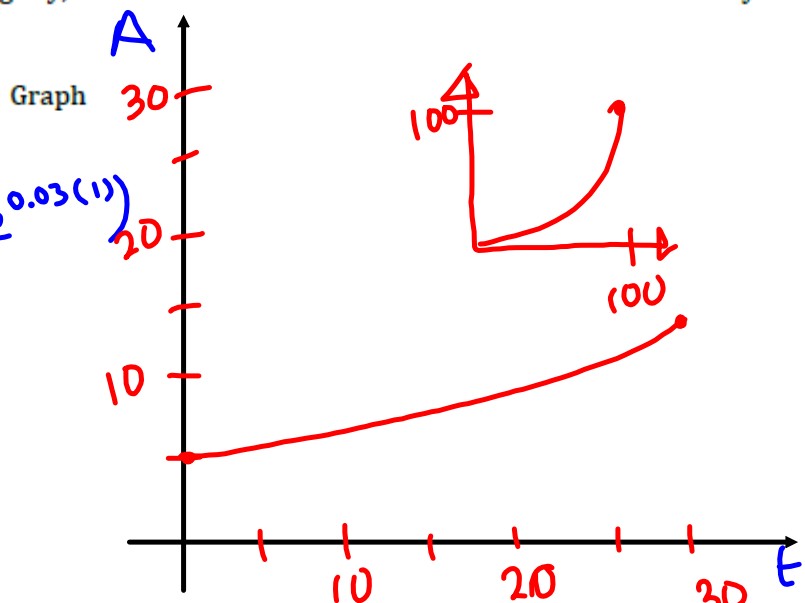
You have been studying a new virus that causes people to break out in spots. Suddenly, a colleague rushes into your office to inform you that there is a confirmed outbreak of the virus in Europe. You know that in the previously-recorded outbreaks, the virus killed around 90% of its victims. The growth of the virus through a population is continuous (until it is somehow contained) at a rate around 3% per day. The current outbreak has 5 confirmed victims.

1. Using this information, create a model of the spread of the spotted virus in this region if it is not contained. To simplify your model slightly, consider the 5 victims as the number of victims on day 0.

Table

days t	victims A
0	5
1	5.15
2	5.31
3	5.47
...	...
10	6.75
20	9.11
30	12.3

Equation



$$A = Pe^{rt}$$

$$A = 5e^{0.03t} \quad \text{or} \quad A(t) = 5e^{0.03t}$$

2. Based on your model, how many people will be infected on:

a) Day 30? $A(30) = 5e^{0.03(30)}$

$$A(30) \approx 12.3 \rightarrow 13 \text{ people}$$

b) Day 60?

$$A(60) \approx 30.25 \rightarrow 31 \text{ people}$$

3. Based upon your model, on what day will there be 50 victims? Show how you arrived at your answer.

$$\frac{50}{5} = \frac{5e^{0.03t}}{5}$$

$$10 = e^{0.03t}$$

$$\ln 10 = \ln e^{0.03t}$$

$$\frac{\ln 10}{0.03} = \frac{0.03t \cdot \ln e}{0.03}$$

$\log_e x = \ln x$
 $\ln e = 1$
 $\log_e e = 1$

day
77 $\leftarrow 76.75 \approx \frac{\ln 10}{0.03} = t$

4. Will the number of days that it will take for the virus to claim 100 victims be double the number of days that it took to claim 50 victims? Why or why not?

5. Calculate the number of days that it will take for the virus to claim 100 victims.

6. On what day will there be 150 victims?

Now you have received a report of a mysterious illness that seems to turn the infected humans into mindless zombies has broken out in a major American city. Since the hungry zombies prey upon innocent people, the outbreak grows continuously at a rate of 12% per day. The outbreak begins with 80 people.

7. How many zombies will there be after 5 days?

8. How many days will it take for the zombie population to reach 3,700,000 (about the population of Los Angeles, CA)?

9. At what rate would the zombie population be growing if it reached 190,000 people (about the population of Salt Lake City, Utah) in 20 days?

Now we're going to get a little more far-fetched in the scenario. Let's say that zombies produce goo that is radioactive and decays continuously with a half-life of 3 years. (That's one more danger of having zombies around.) The half-life tells us that after 3 years, only half of the amount of goo we started with is remaining.

10. If we start with 10 pounds of zombie goo, how much will be remaining after 5 years?

11. How long will it take for the amount of zombie goo to decay to an amount less than 0.5 pounds?

12. When will there be no zombie goo left?

Homework

Finish 2.7 "Ready, Set, Go"