

Questions on 2.4 HW?

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Use the given values and the properties of logarithms to find the indicated logarithm. Do not use a calculator to evaluate the logarithms.

Given: $\log 16 \approx 1.2$
 $\log 5 \approx 0.7$
 $\log 8 \approx 0.9$

10. Find $\log \frac{5}{8} =$
 $\log 5 - \log 8 =$
 $0.7 - 0.9 = \underline{\underline{-0.2}}$

11. Find $\log 25$

12. Find $\log \frac{1}{2}$

13. Find $\log 80$

14. Find $\log \frac{1}{64}$

Given $\log_3 2 \approx 0.6$
 $\log_3 5 \approx 1.5$

15. Find $\log_3 16$

16. Find $\log_3 108$

17. Find $\log_3 \frac{3}{50}$

18. Find $\log_3 \frac{8}{15}$

19. Find $\log_3 486$

$$\text{b.) } \log_2 x - \log_2 5 = 0$$

$$\log_2 x = \log_2 5$$

$$x = 5$$

$$1. 3^{x+4} = 243$$

$$3^{x+4} = 3^5$$

$$x+4 = 5$$

$$4. \log_2 x - \log_2 13 = 0$$

$$x = 13$$

$$2. \left(\frac{1}{2}\right)^x = 8$$

$$3. \left(\frac{3}{4}\right)^x = \frac{27}{64}$$

$$5. \log_2(2x-4) - \log_2 8 = 0$$

$$6. \log_2(x+2) - \log_2 9x = 0$$

$$\log_2(x+2) = \log_2 9x$$

$$x+2 = 9x$$

$$-2 = 8x \quad x = -\frac{1}{4}$$

$$7. \frac{\log 2x}{\log 14} = 1$$

$$8. \frac{\log(5x-1)}{\log 29} = 1$$

$$\log 2x = \log 14$$

$$\log 5^{x-2} = \log 625$$

$$5^{x-2} = 625$$

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$$5^{x-2} = 5^4$$

$$x-2 = 4$$

$$x = 6$$

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Use your calculator and the definition of $\log x$ (recall: the base is 10) to find the value of x .
(Round your answers to 4 decimals.)

23. $\log x = -3$
 $10^{-3} = x$
 $\frac{1}{1000} = x$

24. $\log x = 1$

25. $\log x = 0$

26. $\log x = \frac{1}{2}$

27. $\log x = 1.75$

28. $\log x = -2.2$

29. $\log x = 3.67$

30. $\log x = \frac{3}{4}$

31. $\log x = 6$

$\log_{10} x = y$
 $10^y = x$

2.6H Compounding the Problem

A Develop Understanding Task

Part I: As an enterprising young mathematician, you know that your superior knowledge of mathematics will help you make better decisions about all kinds of things in your life. One important area is money \$\$\$\$. So, you've been contemplating the world and wondering how you could maximize the money that you make in your savings account.



You're young and you haven't saved much money yet. As a matter of fact, you only have \$100, but you really want to make the best of it. You like the idea of compound interest, meaning that the bank pays you interest on all the money in your savings account, including whatever interest that they had previously paid you. This sounds like a very good deal. You even remember that the formula for compound interest is exponential. Let's see, it is:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$0.25\% = 0.0125$$

Where A = the amount of money in the account at any time t

P = the principal, or the original amount invested in the account

r = the annual interest rate (turn % into decimal)

n = the number of compounding periods each year

t = the number of years

1. If your saving account pays a generous 5% per year and is compounded only once each year, how much money would be in the account at the end of one year?

$$\begin{aligned} P &= 100 \\ r &= 0.05 \\ n &= 1 \\ t &= 1 \end{aligned}$$

$$\begin{aligned} A &= 100\left(1 + \frac{0.05}{1}\right)^{1 \cdot 1} \\ A &= 100(1.05) \\ A &= \$105 \end{aligned}$$

2. How much money would be in the account at the end of 20 years?

$$\begin{aligned} P &= 100 \\ r &= 0.05 \\ n &= 1 \\ t &= 20 \end{aligned}$$

$$\begin{aligned} A &= 100\left(1 + \frac{0.05}{1}\right)^{1 \cdot 20} \\ A &= 100(1.05)^{20} \\ A &= \$265.33 \end{aligned}$$

It seems like the more compounding periods in the year, the more money that you should make. The question is, does it make a big difference?

3. Compare the amount of money that you would have after 20 years if it is compounded twice each year (semi-annually), 4 times per year (quarterly), 12 times per year (monthly), 365 times per year (daily), and then hourly. Find a way to organize, display and explain your results to your class.

$$P = \$100$$

$$r = 0.05$$

$$t = 20$$

$$n = 2, 4, 12, 365, 8760$$

semi-annually $n=2$

$$A = 100 \left(1 + \frac{0.05}{2}\right)^{2 \cdot 20} = \$268.51$$

quarterly $n=4$

$$A = 100 \left(1 + \frac{0.05}{4}\right)^{4 \cdot 20} = \$270.15$$

monthly $n=12$

$$A = 100 \left(1 + \frac{0.05}{12}\right)^{12 \cdot 20} = \$271.26$$

daily $n=365$

$$A = 100 \left(1 + \frac{0.05}{365}\right)^{365 \cdot 20} = \$271.81$$

hourly $n=8760$

$$A = 100 \left(1 + \frac{0.05}{8760}\right)^{8760 \cdot 20} = \$271.83$$

It turns out that the value you found in your compounding problem is 100 times a very famous irrational number, named e . Because e is irrational it is a non-terminal, non-repeating decimal number, like π . The first few digits of e are 2.7182818284590452353602874713527. Like π , e is a number that occurs in the mathematics of many real world situations, including exponential growth. One of the formulas using e results from the thinking that you just did about compound interest. It can be shown that the amount of money A in a savings account where money is compounded continuously is given by:

$$A = Pe^{rt}$$

P = the principal, or the original amount invested in the account

r = the annual interest rate

t = the number of years

4. It is fairly typical for savings accounts to be compounded monthly. Compare the amount of money in two savings accounts after 10 years with the same initial investment of \$500 and interest rate of 3% in each account if the first account is compounded monthly and the second account is compounded continuously.

$$\begin{aligned} &\text{monthly} \\ A &= 500 \left(1 + \frac{0.03}{12}\right)^{12 \cdot 10} \\ A &= \$674.68 \end{aligned}$$

$$\begin{aligned} &\text{continuously} \\ A &= 500e^{0.03(10)} \\ A &= \$674.93 \end{aligned}$$

5. Use technology to compare the graphs of the two accounts. What conclusions would you draw about the effect of changing the number of compounding periods on a savings account?

we have more interest gathering the more frequently we are compounding the \$.

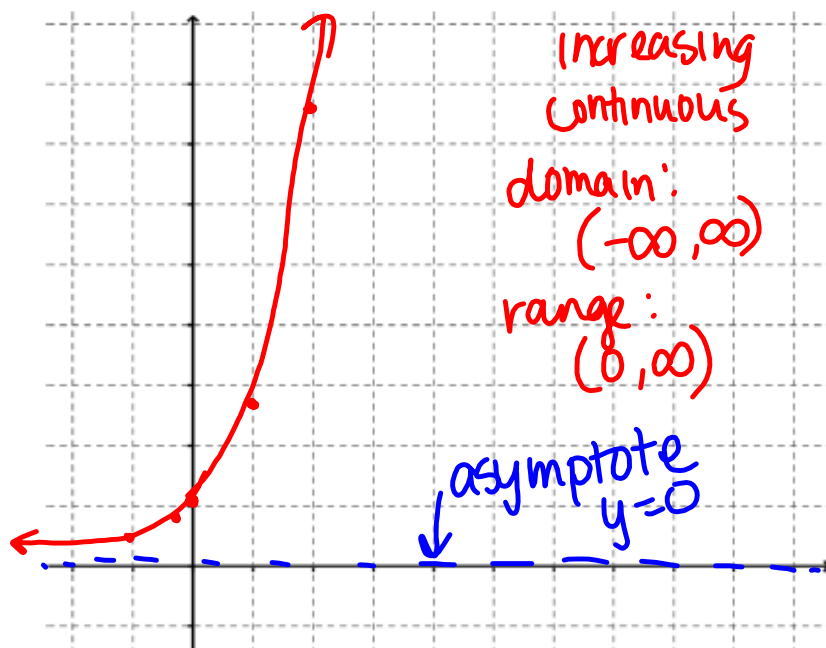
Part II

Since e is widely used to model exponential growth and decay in many contexts, let's get a little more familiar with the base e exponential function:

$$f(x) = e^x$$

1. Make a prediction about the graph of $f(x)$. Explain what knowledge you used to make your prediction.
2. Create a table and a graph and describe the mathematical features of $f(x)$.

x	y
$-\frac{1}{2}$	0.6
$-\frac{1}{4}$	0.78
0	1
1	2.72
2	7.4
3	20.1



Homework

Finish 2.6 "Ready, Set, Go"