Questions on 2.3 homework?

Test Corrections

- Earn 75% of points missed back.
- Due October 4
- Separate piece of paper w/ explanation of mistake made & how you fixed your mistake.

Test Point Breakdown

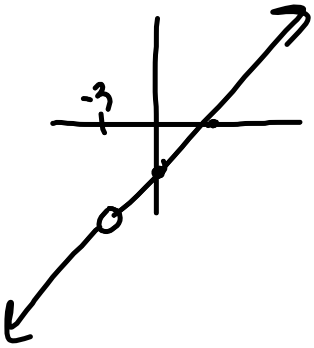
1) 2 pts
2) 3
3) 4
4) 9 → a) 3 pts, b) 3, c) 3
5) 2
6) 3
7) 7 → a) 3, b) 2, c) 2
8) 4 → a) 2, b) 2
9) 4
10) 6 → a) 3, b) 3
11) no #11
12) 1

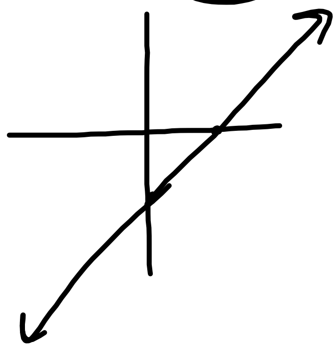
Rebekah Hansen

2.3 HW

25 $f(x)=\frac{x^2-9}{x+3}=\frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}}=x-3$

$x=-3$





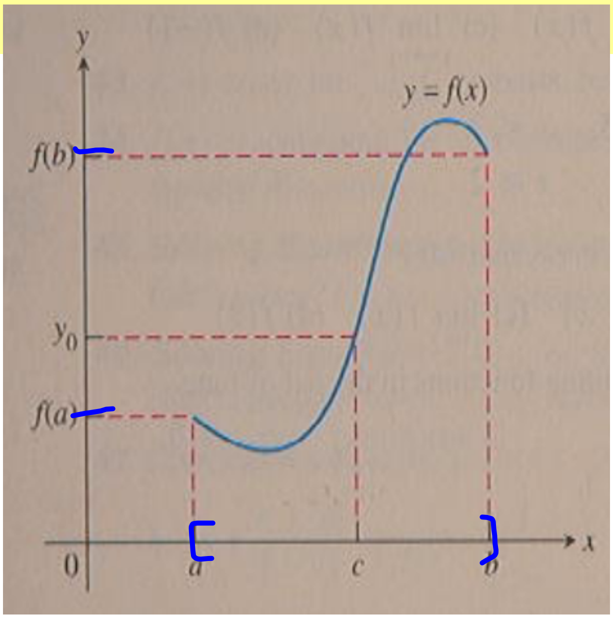
29 $f(x)=\frac{x-4}{\sqrt{x}-2}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}=\frac{(x-4)(\sqrt{x}+2)}{x+\cancel{2\sqrt{x}}-\cancel{2\sqrt{x}}-4}$

$x=4$

$=\frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{(x-4)}}=\boxed{\sqrt{x}+2}$

Intermediate Value Theorem (IVT) for Continuous Functions:

A function $y = f(x)$ that is continuous on a closed interval $[a, b]$ takes on every value between $f(a)$ and $f(b)$. In other words, if y_0 is between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a, b]$.



The Intermediate Value Theorem for Continuous Functions is the reason why the graph of a function continuous on an interval cannot have any breaks. The graph will be **connected**, a single, unbroken curve. It will not have jumps or separate branches.

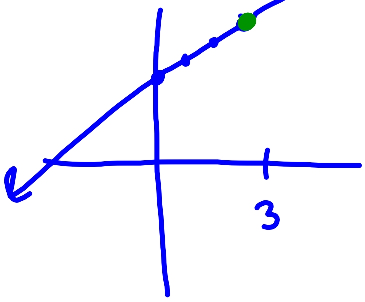
$$\frac{(x+8)\cancel{(x-3)}}{\cancel{(x-3)}} = x+8$$

EXAMPLES:

1. Given $f(x)=\begin{cases} \dfrac{x^2+5x-24}{x-3} & ; \ x\neq 3 \\ k & ; \ x=3 \end{cases}$, find the value for k that will make $f(x)$ continuous for all x.

$3+8=k$

$\boxed{11=k}$



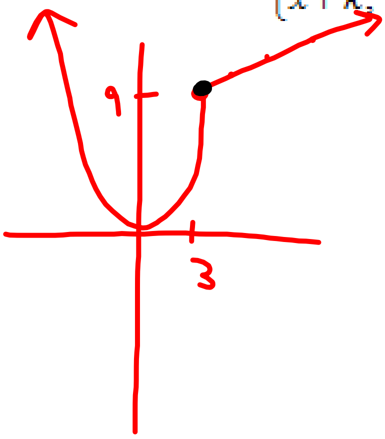
2. Given $g(x)=\begin{cases} x^2 & ; \ x<3 \\ x+k; & x\geq 3 \end{cases}$, what value of k will make $f(x)$ continuous?

$x+6$

$3^2=9$

$3+k=9$

$k=6$

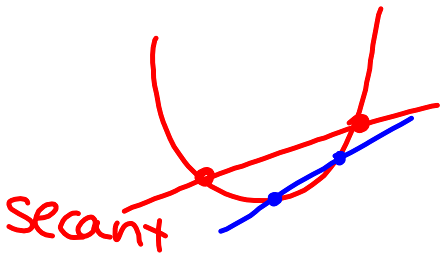


Average Rate of Change

The **average rate of change** of a quantity over a period of time is the amount of change divided by the time it takes.

In general, the ***average rate of change*** of a function over an interval is the amount of change divided by the length of the interval.

Also, the **average rate of change** can be thought of as the slope of a secant line to a curve.



Example:

Find the average rate of change of $f(x) = 2x^2 - 3x + 7$ over the interval $[-2,4]$

$$\frac{f(-2)-f(4)}{-2-4}$$

$$\frac{f(-2)-f(4)}{-2-4} = \frac{\left(2(-2)^2 - 3(-2) + 7\right) - \left(2(4)^2 - 3(4) + 7\right)}{-2-4}$$

$$= \frac{21-27}{-6} = \frac{-6}{-6} = 1$$

Tangent to a Curve

In calculus, we often want to define the rate at which the value of a function $y = f(x)$ is changing with respect to x at any particular value $x = a$ to be the slope of the tangent to the curve $y = f(x)$ at $x = a$.

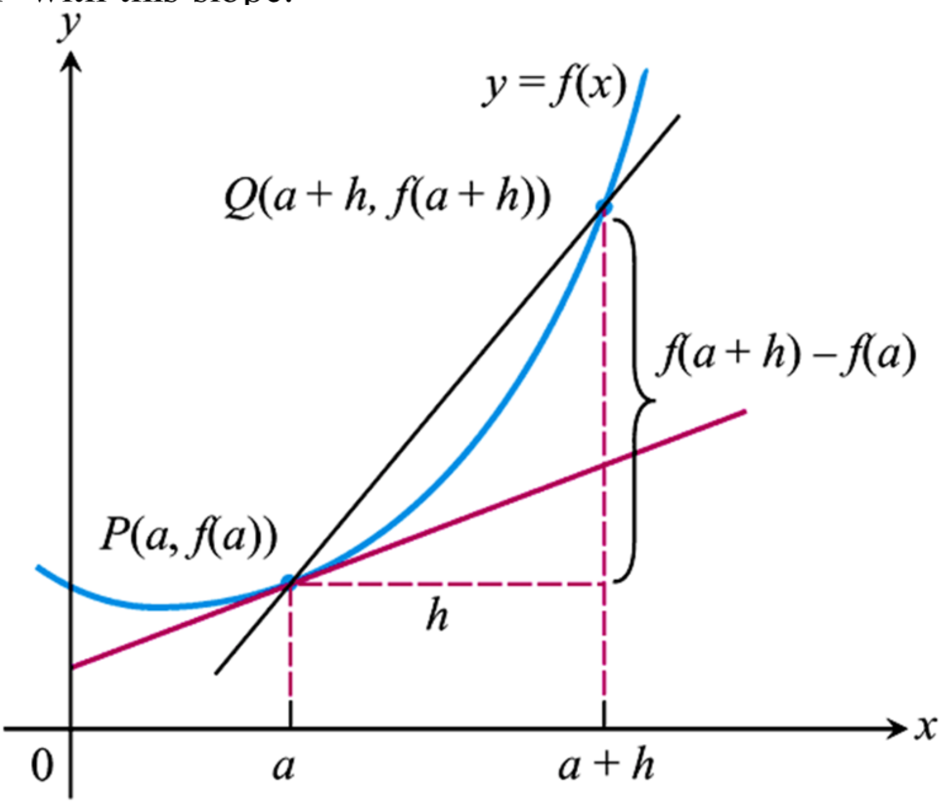
The problem with this is that we only have one point and our usual definition of slope requires two points.

The process becomes:

1. Start with what can be calculated, namely, the slope of a secant through P and a point Q nearby on the curve.

2. Find the limiting value of the secant slope (if it exists) as Q approaches P along the curve.

3. Define the *slope of the curve* at P to be this number and *define the tangent to the curve* at P to be the line through P with this slope.



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Example:

Given $y=x^2+2$ at $x=-1$ find:
the slope of the curve and an equation of the tangent line.
Then draw a graph of the curve and tangent line in the same viewing window.

(a) Write an expression for the slope of the secant line and find the limiting value of the slope as Q approaches P along the curve.

$(-1+h, y(-1+h))$

When $x=-1$, $y=x^2+2=3$ so $P(-1,3)$

$$\lim_{h\to 0}\frac{y(-1+h)-y(-1)}{(-1+h)+(+1)}=\lim_{h\to 0}\frac{(-1+h)^2+2-\left[(-1)^2+2\right]}{h}$$

$$\lim_{h\to 0}\frac{\cancel{3}-2h+h^2-\cancel{3}}{h}=\lim_{h\to 0}\frac{h^2-2h}{h}=\lim_{h\to 0}\frac{\cancel{h}(h-2)}{\cancel{h}}=\lim_{h\to 0}(h-2)=-2$$

$f(0)=0-2$

$(-1+h)(-1+h) \Rightarrow 1-2h+h^2+2=3-2h+h^2$

(b) The tangent line has slope -2 and passes through $(-1,3)$.
The equation of the tangent line is

$y-y_1=m(x-x_1)$

$y-3=-2(x-(-1))$

$y=-2(x+1)+3$

$y=-2x-2+3$

$y=-2x+1$

$y=mx+b$

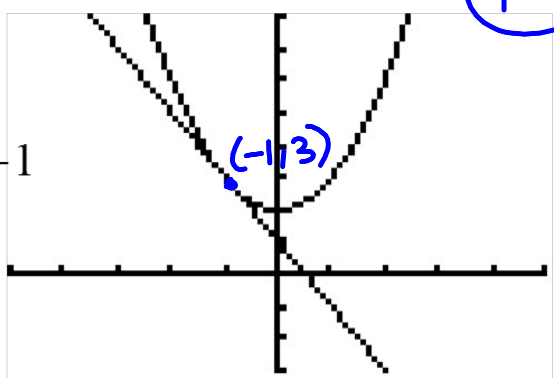
$3=-2(-1)+b$

$3=2+b$

$1=b$

tangent

$y=-2x+1$



Slope of a Curve at a Point:

The **slope of the curve** $y=f(x)$ at the point $P(a, f(a))$ is the number

$$m=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

provided the limit exists.

The **tangent line to the curve** at P is the line through P with this slope.

FYI All of the following mean the same:

1. the slope of $y = f(x)$ at $x = a$
2. the slope of the tangent to $y = f(x)$ at $x = a$
3. the (instantaneous) rate of change of $f(x)$ with respect to x at $x = a$
4. $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

The expression $\frac{f(a+h)-f(a)}{h}$ is the **difference quotient** of f at a.

Normal to a Curve:

The **normal line** to a curve at a point is the line perpendicular to the tangent at the point.

The slope of the normal line is the negative reciprocal of the slope of the tangent line.

Example:

Given $y=x^2+2$ at $x=-1$ write the equation of the normal line. Draw a graph of the curve, the tangent line and the normal line in the same viewing window.

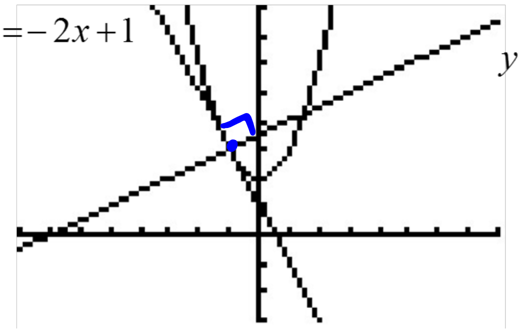
From an earlier example, the slope of the tangent line was found to be -2 so the slope of the normal is $\frac{1}{2}$.

$$y-3=\frac{1}{2}\left(x-(-1)\right)$$

$$y=\frac{1}{2}(x+1)+3$$

$$y=\frac{1}{2}x+\frac{1}{2}+\frac{6}{2}$$

$$y=\frac{1}{2}x+\frac{7}{2}$$



Speed

The function $y = 16t^2$ is an object's **position function**. An object's average speed along a coordinate axis for a given period of time is the average rate of change of its position $y = f(t)$.

Its **instantaneous speed** at any time t is the **instantaneous rate of change** of position with respect to time at time t, or $\lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$.

Homework

2.4 pg.92-93 #3-30(X3)

Oct 21-8:47 AM