

## Questions on lesson 2.2?

We will be having our concept mastery quiz shortly.

# Sleep Tight

## 2.3

### Using Confidence Intervals to Estimate Unknown Population Means

PG.72 IN YOUR BOOK

#### PROBLEM 1 Every Vote Counts: Exploring Categorical Data



In a poll of 1100 registered voters before an upcoming mayoral election, 594 people, or 54%, said they would vote to re-elect the current mayor, while the remaining voters said they would not vote for the mayor. The margin of error for the poll was  $\pm 3$  percent, which means that the poll predicts that somewhere between 51% ( $54\% - 3\%$ ) and 57% ( $54\% + 3\%$ ) of people will actually vote to re-elect the mayor.

The poll results are categorical data because there are two categories: those who will vote for the mayor and those who won't.



1. Does the poll represent a sample survey, an observational study, or an experiment?



PG.73 IN YOUR BOOK

4. With your classmates, conduct a simulation to represent polling a new sample of 1100 voters.

a. Divide 1100 by the number of students in your class to determine the size of each student's sample.

$$\frac{1100}{36} = 30.\bar{5} = \boxed{31}$$

randInt(1,100,31)  
randInt(1,100)

b. Generate an amount of random numbers equal to the sample size in part (a) to represent responses to the polling question. Generate random numbers between 1 and 100, with numbers from 1 to 54 representing support for re-electing the mayor and the numbers 55 to 100 representing support for not re-electing the mayor. Tally the results of your simulation, and then list the total number of tallies for each category.

Here we assume that an average of 54% will vote to re-elect the mayor.



Number of People Who Respond that They Will Vote to Re-elect the Mayor	Number of People Who Respond that They Will Not Vote to Re-elect the Mayor
20 19 19 17 13	14 14 14 15 11
21 15 22 19 17	12 13 13 13 11
18 17 20 16 18	15 11 12 11 10
20 14 17 21 20	10 12 16 17 10
21 19 17 16 20	15 12 16 17 10
13 16 17 16 22	9 9 14 18 10
17 16 18 22 22	9 11 17 14 10
	18 14 17 14 10
$\boxed{560}$	$\boxed{410}$

PG.74 IN YOUR BOOK

c. Calculate the percent of people who state that they will vote to re-elect the mayor and the percent of people who state that they will vote to not re-elect the mayor based on your simulation.

d. Complete the simulation for the 1100 voters by combining the data from your classmates. List the percent of votes for each category.

Percent of People Who Respond that They Will Vote to Re-elect the Mayor	Percent of People Who Respond that They Will Vote to Not Re-elect the Mayor

PG.75 IN YOUR BOOK

The percent of voters who actually vote for the mayor in the election is the **population proportion**. The percent of voters in the sample who respond that they will vote for the mayor is the **sample proportion**. The population proportion and sample proportion are measures used for discrete, or categorical, data. For continuous data, these are called the population mean and sample mean.

For continuous data, it's called the population or sample mean. For categorical data, it's called the population or sample proportion.



When you and your classmates generated random numbers to simulate multiple samples of the ~~1100~~ <sup>970</sup> voters, you came up with different sample proportions. The set of all of your classmates' sample proportions is part of a *sampling distribution*.

A **sampling distribution** is the set of sample proportions for all possible equal-sized samples. A **sampling distribution** will be close to a normal distribution, and the **center of a sampling distribution** is a good estimate of a **population proportion**—in this case, the percent of people who will actually vote to re-elect the mayor.

You can learn the details of deriving the formula for the standard deviation of the sampling distribution,  $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ , in a statistics course.

But rather than collecting a very large number of samples, a more practical method for estimating a population proportion is to use the sample proportion of a single sample to estimate the standard deviation of the sampling distribution. The **standard deviation of a sampling distribution** can give you a range in which the population proportion is likely to fall, relative to the sample proportion.



For example, to estimate the standard deviation of the sampling distribution for the sample of 1100 voters, you can use the formula  $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ , where ( $\hat{p}$ ) is the sample proportion and  $n$  is the sample size.

$\hat{p}$   
"p-hat"

$\frac{594}{1100}$

The sample proportion from the original poll is 54%, or 0.54. This is the percent of the 1100 people in the poll who said they would vote to re-elect the mayor.

The standard deviation of the sampling distribution for this poll is

$$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.54(1-0.54)}{1100}}$$

$$\approx 0.0150 \approx 1.5\%$$

This means that 1 standard deviation below the sample proportion of 54% is 54% - 1.5%, or 52.5%. And 1 standard deviation above the sample proportion of 54% is 54% + 1.5%, or 55.5%.

## PG.77 IN YOUR BOOK

An estimated range of values that will likely include the population proportion or population mean is called a confidence interval. When stating the margin of error, a 95% confidence interval is typically used. However, other confidence intervals may also be used.

For example, the standard deviation of the sampling distribution for the election sample is 0.015, or 1.5%. Two standard deviations is 3%, so the margin of error is reported as  $\pm 3\%$ .

Confidence intervals for a population proportion are calculated using the sample proportion of a sample and the standard deviation of the sampling distribution.

- The lower bound of a 68% confidence interval ranges from 1 standard deviation below the sample proportion to 1 standard deviation above the sample proportion.
- The lower bound of a 95% confidence interval ranges from 2 standard deviations below the sample proportion to 2 standard deviations above the sample proportion.
- The lower bound of a 99.7% confidence interval ranges from 3 standard deviations below the sample proportion to 3 standard deviations above the sample proportion.

$$54 \pm 2(1.5) = 54 \pm 3$$

NOT IN YOUR BOOK

1. Roosevelt High School is considering a requirement for all 1300 student to wear uniforms to school. Of the 300 parents surveyed by the school, 141 said they were in favor of mandatory school uniforms.

a. Determine the sample proportion that represents the percent of parents in favor of mandatory school uniforms.

$$\frac{141}{300} = 0.47 = 47\%$$

b. Determine a 95% confidence interval for the population proportion using the sample proportion you determined in part a. Round your answer to the nearest tenth of a percent.

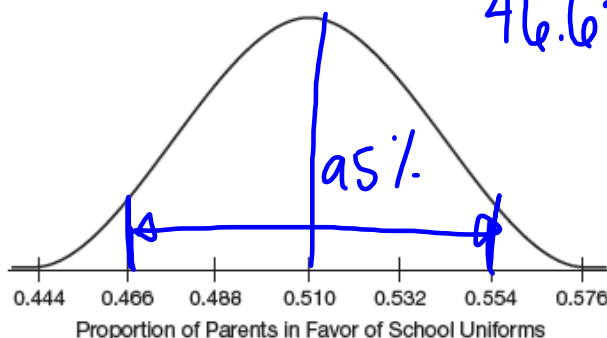
$$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.47(1-0.47)}{300}} \approx 0.029$$

$\hat{p} = 0.47$   
 $n = 300$

95% C.I. 2.9%

$$47\% \pm 2(2.9) \rightarrow 41.2\% \text{ to } 52.8\%$$

c. A group of high school students conducted their own survey of 500 parents. The normal curve displays the sample proportion that represents the percent of parents in favor of mandatory school uniforms and the standard deviation of the sampling distribution. Determine a 95% confidence interval for the population proportion based on the students' sample proportion. Round your answer to the nearest tenth of a percent.



d. Based on the given information in part c, how many of the parents who responded to the students' survey are in favor of mandatory school uniforms?

e. Which survey would you expect to be more accurate, the school's survey or the students' survey? Explain your reasoning.

continuous data

$$\frac{s}{\sqrt{n}}$$

through pg. 79  
try pg. 80-81

## NOT IN YOUR BOOK

2. Two hundred teenage boys were surveyed about the number of hours they spend each week playing video games. The sample mean was 11.7 hours and the standard deviation was 3.4 hours.
  - a. Determine the standard deviation for the population mean.
  
  - b. Determine a 95% confidence interval for the population mean.
  
3. Five hundred teenage girls were surveyed about the number of hours they spend each week listening to music. The sample mean was 9.2 hours and the standard deviation was 2.7 hours.
  - a. Determine the standard deviation for the population mean.
  
  - b. Determine a 95% confidence interval for the population mean.

# Homework

Finish lesson 2.3 through pg.81