Questions on your homework? Our limits quiz will be soon...

1 - 2

2 - 2.75

3 - -6

4 - DNE

5 - $\frac{5}{3}$

6 - $-\infty$

7 - 3

8 - $a = -\frac{1}{3}$; b - $\frac{5}{3}$

9 - 0

10 - ∞

11 - $\frac{1}{3}$

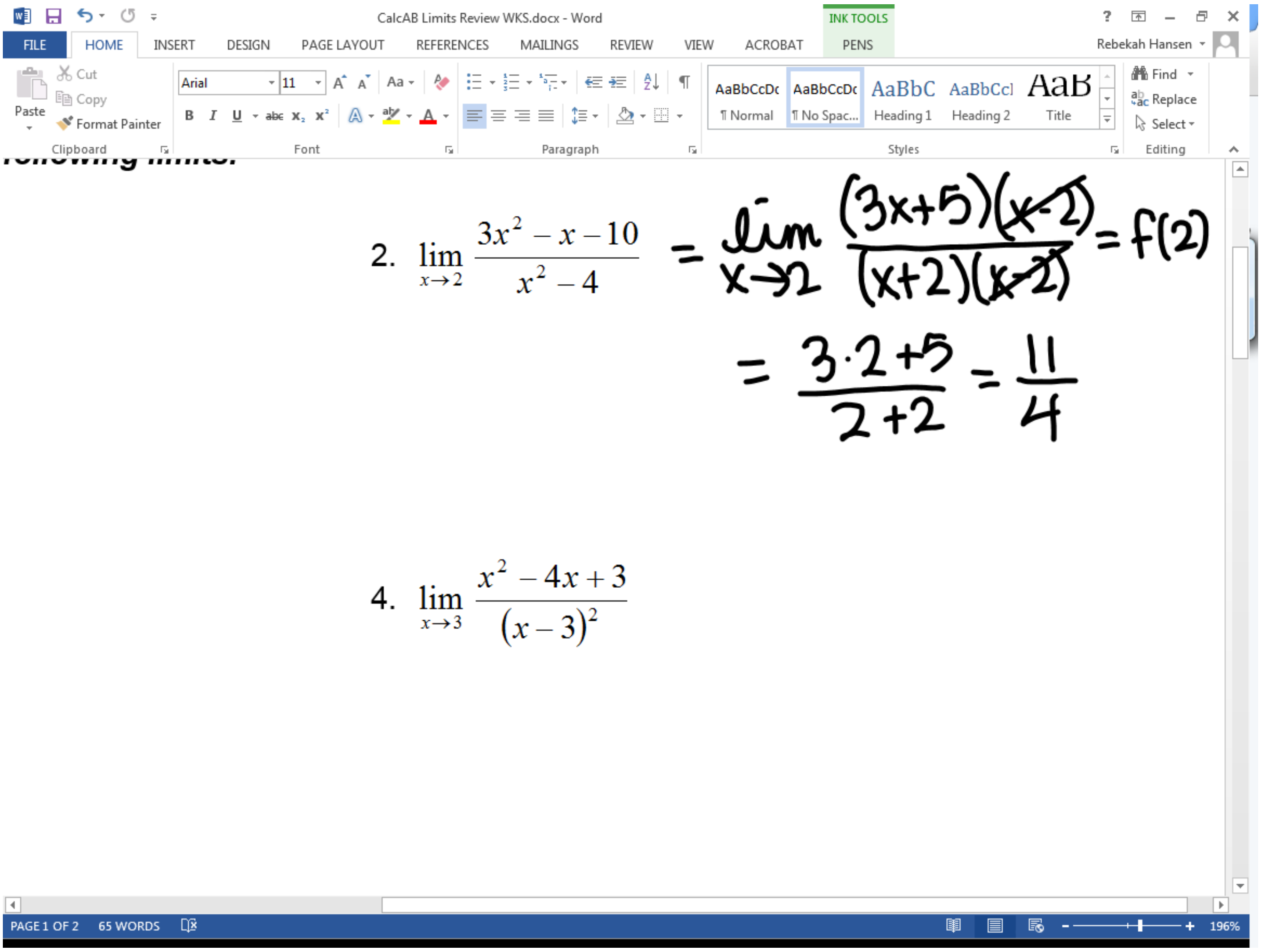
12 - 0

13 - 0

14 - $\frac{1}{3}$

15 - 0

16 - ∞



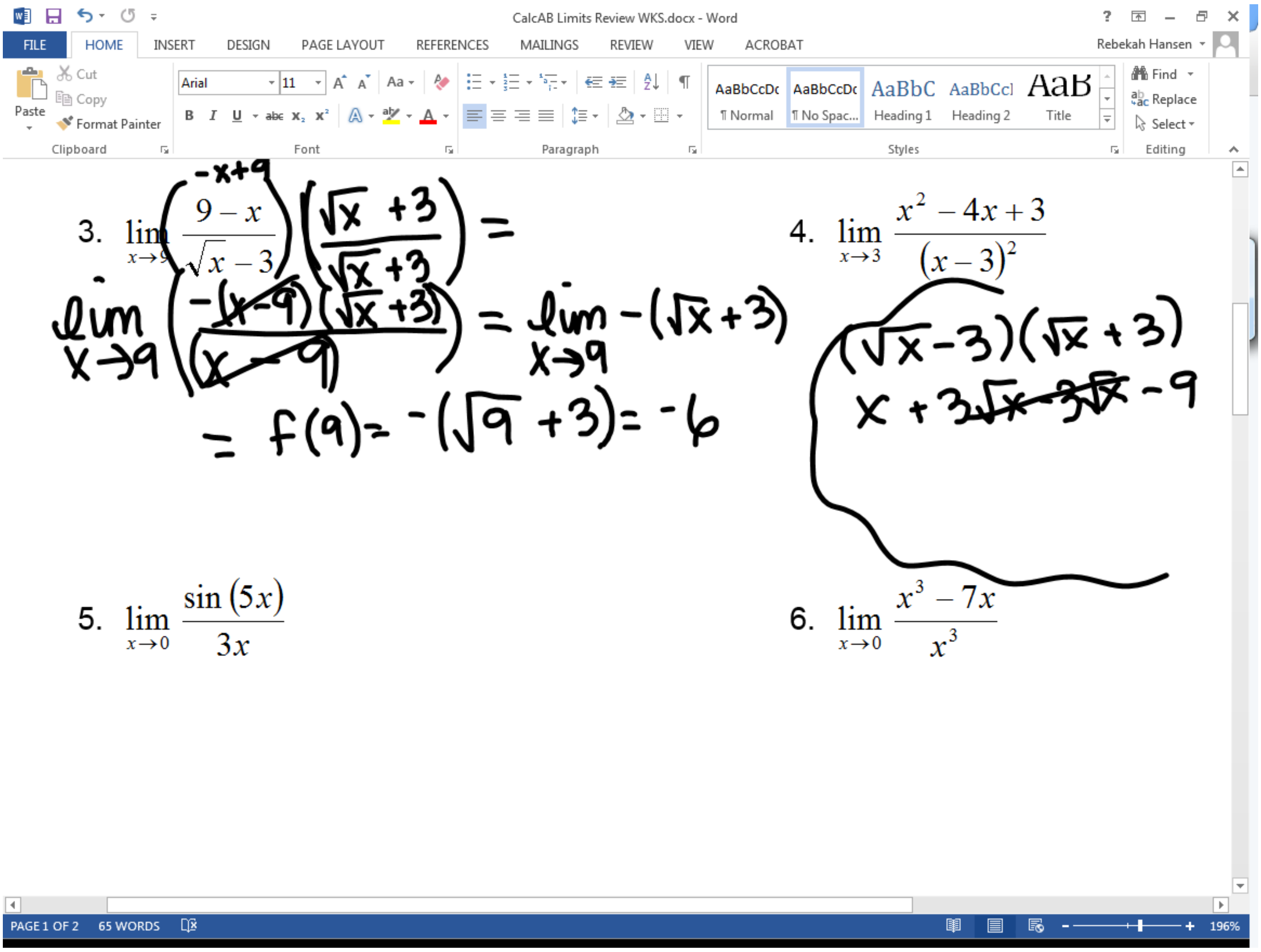
following limits:

2. $\lim\limits_{x\to 2} \dfrac{3x^2 - x - 10}{x^2 - 4}$ $= \lim\limits_{x\to 2} \dfrac{(3x+5)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = f(2)$

$= \dfrac{3\cdot 2+5}{2+2} = \dfrac{11}{4}$

4. $\lim\limits_{x\to 3} \dfrac{x^2 - 4x + 3}{(x-3)^2}$

Sep 23-9:27 AM



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3. $\lim\limits_{x\to 9}\left(\dfrac{\overset{-x+9}{9-x}}{\sqrt{x}-3}\right)\left(\dfrac{\sqrt{x}+3}{\sqrt{x}+3}\right)=$

$\lim\limits_{x\to 9}\left(\dfrac{-\cancel{(x-9)}(\sqrt{x}+3)}{\cancel{(x-9)}}\right)=\lim\limits_{x\to 9}-(\sqrt{x}+3)$

$=f(9)=-(\sqrt{9}+3)=-6$

4. $\lim\limits_{x\to 3}\dfrac{x^2-4x+3}{(x-3)^2}$

$(\sqrt{x}-3)(\sqrt{x}+3)$

$x+\cancel{3\sqrt{x}-3\sqrt{x}}-9$

5. $\lim\limits_{x\to 0}\dfrac{\sin(5x)}{3x}$

6. $\lim\limits_{x\to 0}\dfrac{x^3-7x}{x^3}$

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4. $\lim_{x\to3}\frac{x^2-4x+3}{(x-3)^2} = \lim_{x\to3}\left(\frac{\cancel{(x-3)}(x-1)}{\cancel{(x-3)}(x-3)}\right) =$

$\lim_{x\to3}(x-1)\cdot\lim_{x\to3}\left(\frac{1}{x-3}\right) = 2\cdot\lim_{x\to3}\left(\frac{1}{x-3}\right) = \underline{\underline{DNE}}$

$\to$ DNE

$\lim_{x\to3^+} f(x) = \infty$

$\lim_{x\to3^-} f(x) = -\infty$

6. $\lim_{x\to0}\frac{x^3-7x}{x^3}$

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8. Consider the function $f(x) = \begin{cases} 3, & x = 2 \\ b - ax^2, & x < 2 \end{cases}$ $f(2) = 3$

Determine the values of constants a and b so that $\lim_{x \to 2} f(x)$ exists and is equal to $f(2)$.

$$\begin{cases} a + b(2) = 3 \\ b - a(2)^2 = 3 \end{cases}$$

$2b + a = 3$

$a = 3 - 2b$

$b - 4a = 3$

$b - 4(3 - 2b) = 3$

$b - 12 + 8b = 3$

$9b - 12 = 3$

$9b = 15$

$b = 15/9 = \frac{5}{3}$

$a + \frac{5}{3}(2) = 3$

$a + \frac{10}{3} = 3$

$a = -1/3$

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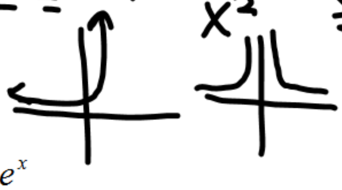
13. $\lim_{x \to -\infty} \frac{7^x}{4 - x^2} = \lim_{x \to -\infty} \left(\frac{7^x}{-x^2+4} \right) =$

$-1(x^2-4)$

$-1\left(\lim_{x \to -\infty} \left(\frac{7^x}{x^2-4} \right)\right) =$

$\to 0 \quad \to 0$

ebm: $\frac{7^x}{x^2} = 7^x \cdot \frac{1}{x^2} = 0$



14. $\lim_{x \to \infty} \frac{x+3}{\sqrt{9x^2 - 5x}}$

15. $\lim_{x \to -\infty} \frac{e^x}{4 + 5e^{3x}}$

16. $\lim_{z \to -\infty} \frac{4x^3 - 3x^2 + 7}{10x^2 - 5x}$

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11. $\lim\limits_{x \to -\infty} \dfrac{x+7}{3x+5}$

ebm: $\dfrac{x}{3x} = \dfrac{1}{3}$

12. $\lim\limits_{x \to \infty} \dfrac{x^2}{x^3+10x-4}$

13. $\lim\limits_{x \to -\infty} \dfrac{7^x}{4-x^2} = \lim\limits_{x \to -\infty}\left(\dfrac{7^x}{-x^2+4}\right) =$

$-1(x^2-4)$

$-1\left(\lim\limits_{x \to -\infty}\left(\dfrac{7^x}{x^2-4}\right)\right) =$

$\to 0 \quad \to 0$

ebm: $\dfrac{7^x}{x^2} = 7^x \cdot \dfrac{1}{x^2} = 0$

14. $\lim\limits_{x \to \infty} \dfrac{x+3}{\sqrt{9x^2-5x}}$

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13. $\lim_{x\to-\infty} \frac{7^x}{4-x^2} = \lim_{x\to-\infty}\left(\frac{7^x}{-x^2+4}\right) =$

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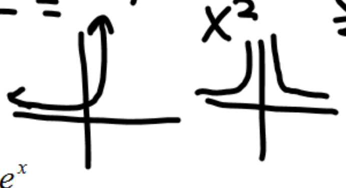
$\to 0 \quad \to 0$

ebm: $\frac{7^x}{x^2} = 7^x \cdot \frac{1}{x^2} = 0$



14. $\lim_{x\to\infty} \frac{x+3}{\sqrt{9x^2-5x}}$

ebm: $\frac{x}{\sqrt{9x^2}} = \frac{x}{3x} = \frac{1}{3}$

15. $\lim_{x\to-\infty} \frac{e^x}{4+5e^{3x}}$

16. $\lim_{z\to-\infty} \frac{4x^3-3x^2+7}{10x^2-5x}$

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Limits Quiz

2) (a) $\lim_{x \to 2^-} f(x)$ (b) $\lim_{x \to 2^+} f(x)$

3) (a) left (b) right

Sep 22-8:24 AM

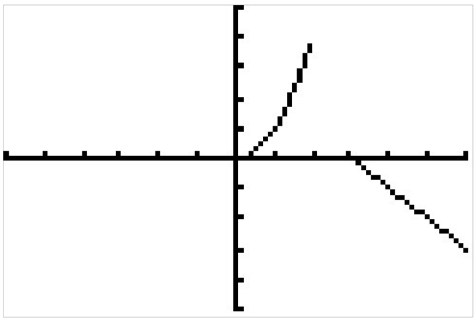
2.3 Continuity

A function is continuous if . . .

Any function $y = f(x)$ whose graph can be sketched in one continuous motion without lifting the pencil is an example of a continuous function.

Example

Find the points at which the given function is continuous and the points at which it is discontinuous.



Points at which f is continuous

At $x=0$ $\qquad \lim_{x\to 0^+} f(x)=f(0)$

At $x=6$ $\qquad \lim_{x\to 6^-} f(x)=f(6)$

At $0<c<6$ but not $2\le c<3$ $\qquad \lim_{x\to c} f(x)=f(c)$

Points at which f is discontinuous

At $x=2$ $\qquad \lim_{x\to 2} f(x)$ does not exist

At $c<0,\ 2<c<3,\ c>6$ $\qquad$ these points are not in the domain of f

Sep 22-8:30 AM

Continuity at a Point:

Interior Point: A function $y=f(x)$ is continuous at an interior point c of its domain if $\lim_{x\to c} f(x)=f(c)$

Endpoint: A function $y=f(x)$ is continuous at a left endpoint a or is continuous at a right endpoint b of its domain if

$\lim_{x\to a^+} f(x)=f(a)$ or $\lim_{x\to b^-} f(x)=f(b)$ respectively.

$x \to a^+$ $x \to b^-$

If a function f is **not continuous at a point** c, we say that f is **discontinuous at** c and c is a point of discontinuity of f.

Note that c need not be in the domain of f.

Sep 22-8:31 AM

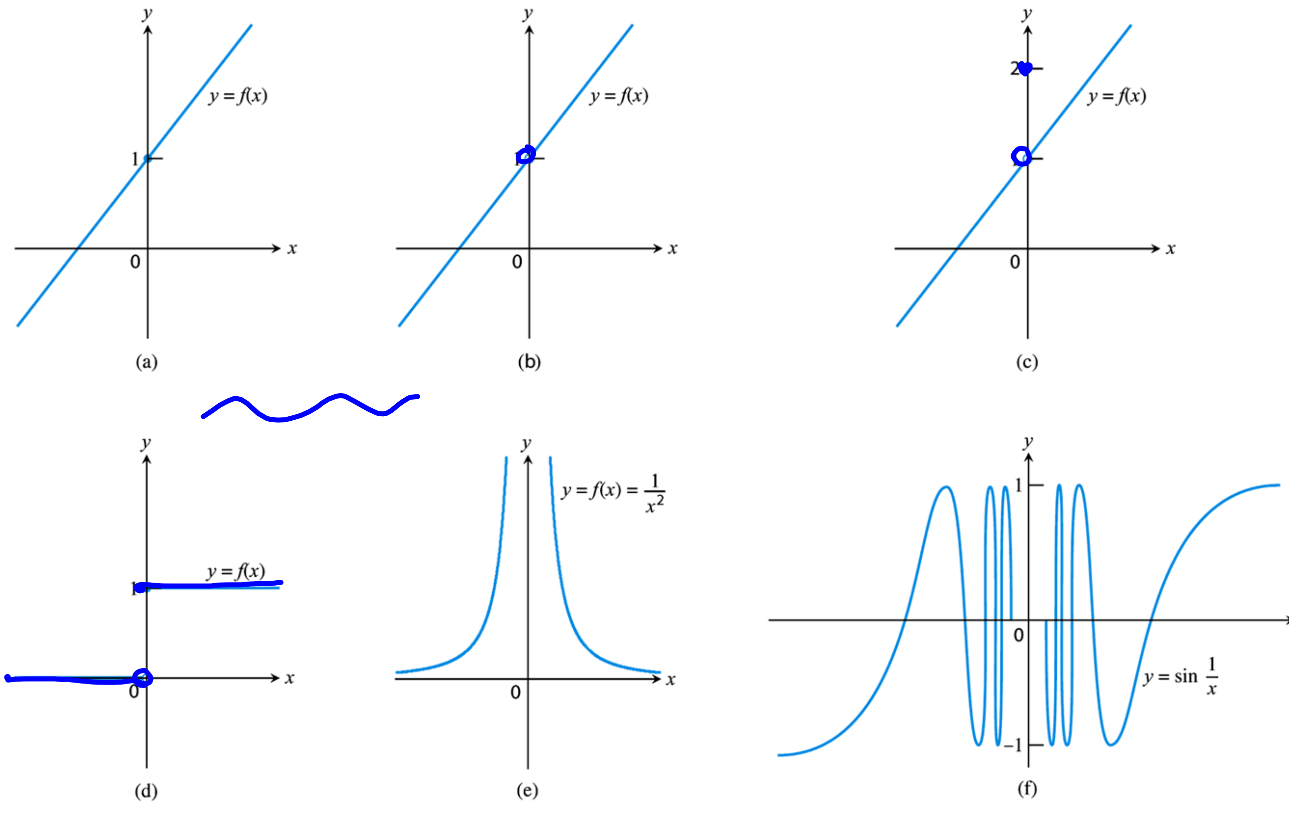
Pg.80 The typical discontinuity types are:

a) Removable (2.21b and 2.21c)

b) Jump (2.21d)

c) Infinite (2.21e)

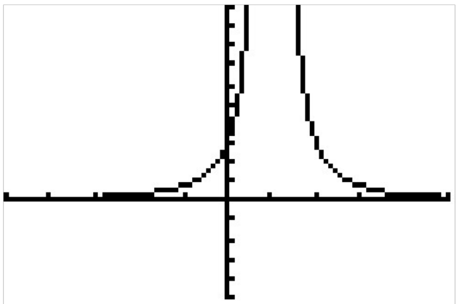
d) Oscillating (2.21f)



Sep 22-8:34 AM

Example:

Find and identify the points of discontinuity of $y = \dfrac{3}{(x-1)^2}$



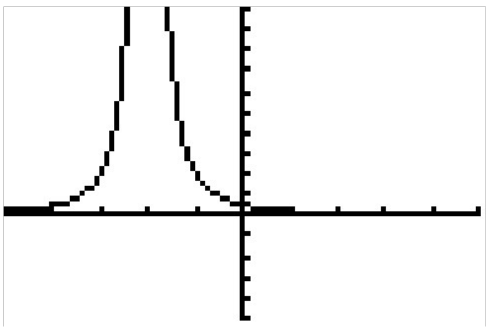
There is an infinite discontinuity at $x = 1$.

Continuous Functions

A function is **continuous on an interval** if and only if it is continuous at every point of the interval. A **continuous function** is one that is continuous at every point of its domain. A continuous function need not be continuous on every interval.

Example:

The given function is a continuous function because it is continuous at every point of its domain. It does have a point of discontinuity at $x=-2$ because it is not defined there.



$$y=\frac{2}{(x+2)^2}$$

Sep 22-8:35 AM

Properties of Continuous Functions (pg.82):

If the functions f and g are continuous at $x = c$, then the following combinations are continuous at $x = c$.

1. Sums: $f + g$
2. Differences: $f - g$
3. Products: $f \cdot g$
4. Constant multiples: $k \cdot f$, for any number k
5. Quotients: $\frac{f}{g}$, provided $g(c) \neq 0$

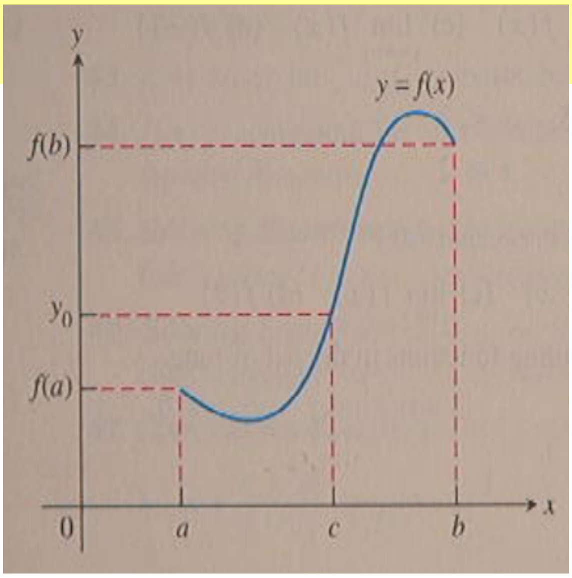
Composition of Continuous Functions:

If f is continuous at c and g is continuous at $f(c)$, then the composite $g \circ f$ is continuous at c.

Sep 22-8:37 AM

Intermediate Value Theorem (IVT) for Continuous Functions:

A function $y = f(x)$ that is continuous on a closed interval $[a,b]$ takes on every value between $f(a)$ and $f(b)$. In other words, if y_0 is between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a,b]$.



The Intermediate Value Theorem for Continuous Functions is the reason why the graph of a function continuous on an interval cannot have any breaks. The graph will be **connected**, a single, unbroken curve. It will not have jumps or separate branches.

EXAMPLES:

1. Given $f(x)=\left\{\begin{matrix} \dfrac{x^2+5x-24}{x-3} & ; & x\neq 3 \\ k & ; & x=3 \end{matrix}\right\}$, find the value for k that will make $f(x)$ continuous for all x.

2. Given $g(x)=\left\{\begin{matrix} x^2 & ; & x<3 \\ x+k; & & x\geq 3 \end{matrix}\right\}$, what value of k will make $f(x)$ continuous?

Homework

2.3 pg.84-85 #1-43 EOO

Oct 21-8:47 AM