2.2 Limits Involving Infinity

REVIEW: Find the following limits.

1. $\lim_{x\to 0} \frac{\sin(3x)}{x}$ $= f(0) = \frac{\sin(3\cdot 0)}{0} = \frac{0}{0} \rightarrow \surd$ graph

$\lim_{x\to 0} \frac{\sin(3x)}{x} = 3$

2. $\lim_{x\to 0} \frac{\sin(5x)}{\sin(4x)}$ $\rightarrow \surd$ graph $\rightarrow \lim_{x\to 0} \frac{\sin(5x)}{\sin(4x)} = 1.25$

3. $\lim_{x\to 0} \frac{1-\cos^2 x}{x}$ $\rightarrow \surd$ graph $\rightarrow \lim_{x\to 0} \frac{1-\cos^2 x}{x} = 0$

$1-\cos^2 x = \sin^2 x$

Questions on 2.1 HW?

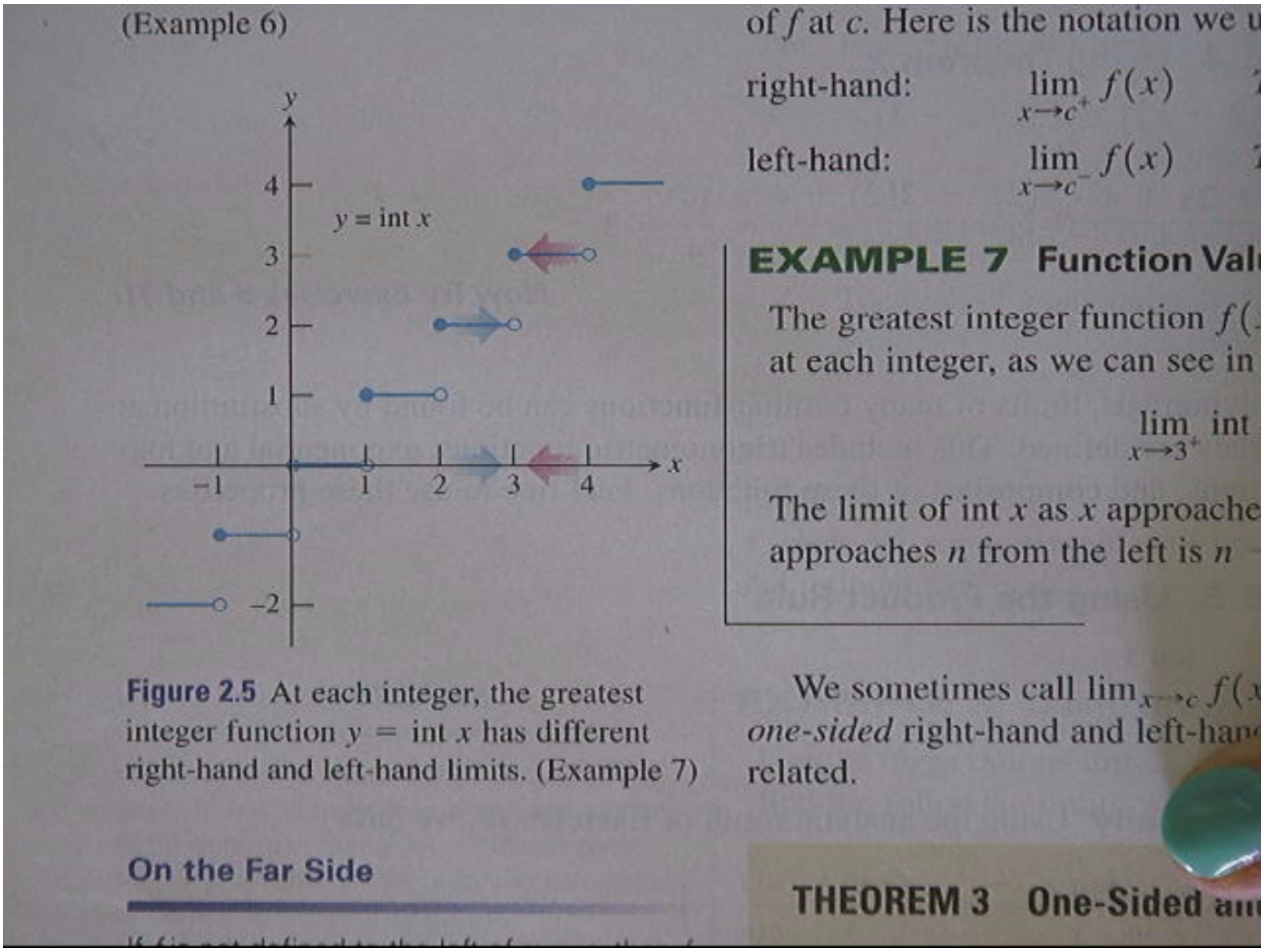

(Example 6)

Figure 2.5 At each integer, the greatest integer function $y = \text{int } x$ has different right-hand and left-hand limits. (Example 7)

On the Far Side

of f at c. Here is the notation we u

right-hand: $\lim_{x \to c^+} f(x)$

left-hand: $\lim_{x \to c^-} f(x)$

EXAMPLE 7 Function Val

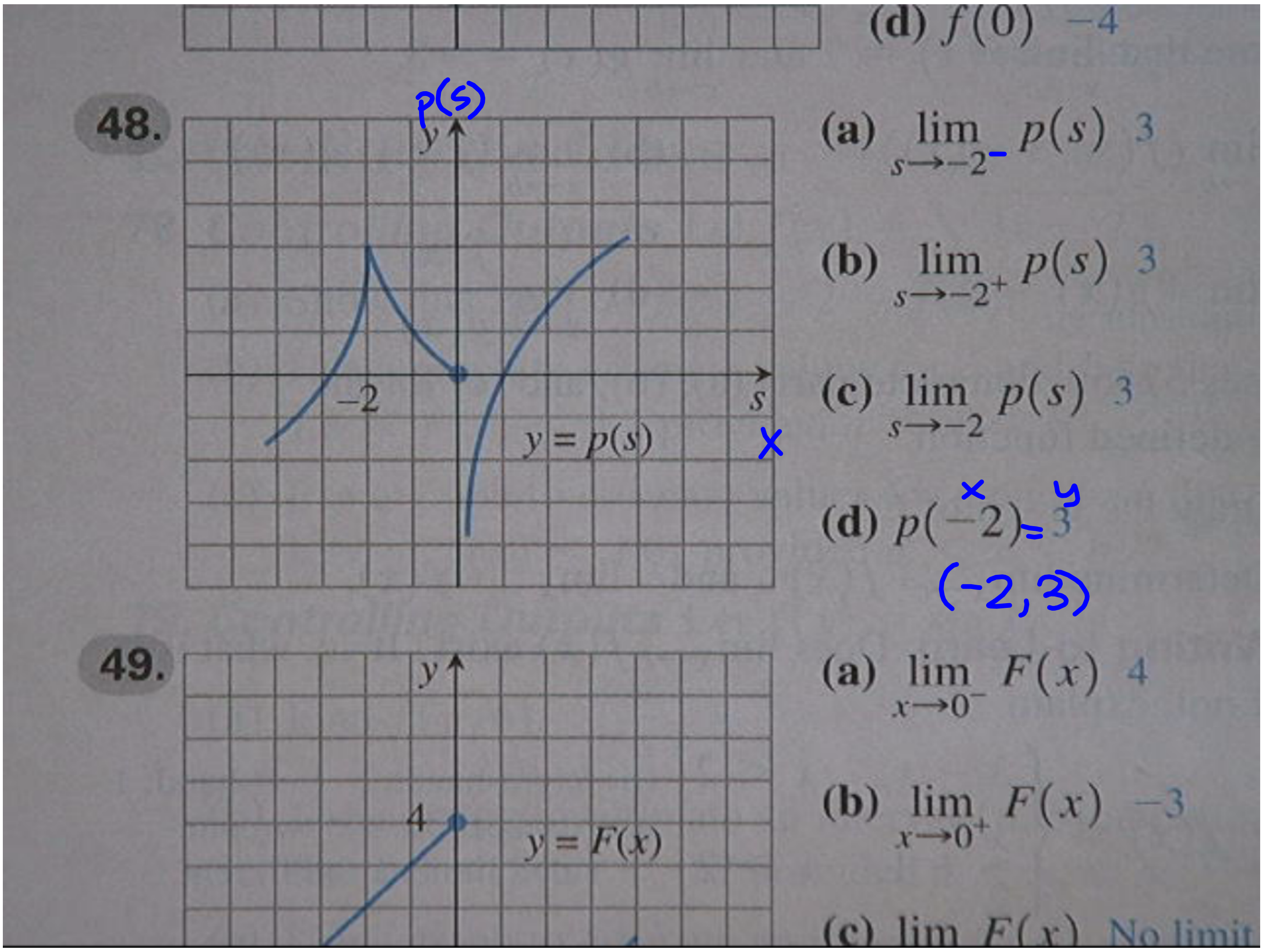
The greatest integer function $f($
at each integer, as we can see in

$\lim_{x \to 3^+} \text{int}$

The limit of int x as x approache
approaches n from the left is n

We sometimes call $\lim_{x \to c} f($
one-sided right-hand and left-han
related.

THEOREM 3 One-Sided an



(d) $f(0)$ −4

48.

(a) $\lim_{s \to -2^-} p(s)$ 3

(b) $\lim_{s \to -2^+} p(s)$ 3

(c) $\lim_{s \to -2} p(s)$ 3

(d) $p(-2) = 3$

(−2, 3)

49.

(a) $\lim_{x \to 0^-} F(x)$ 4

(b) $\lim_{x \to 0^+} F(x)$ −3

(c) lim $F(x)$ No limit

Function(F) View(V) Option(O) Help(H)

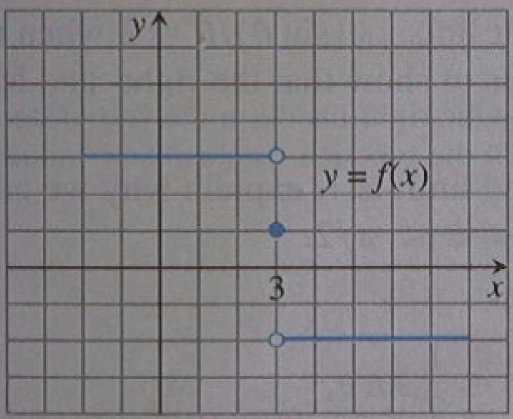
SXGA XGA VGA 50% 100% Fit Strch Full Qsav Still Set Rec Stop Set Ctrl Panel Tool Bar VGA 30FPS 00:00:00

(h) $\lim_{x\to c} f(x)$ exists at every c in $(-1, 1)$. True

(i) $\lim_{x\to c} f(x)$ exists at every c in $(1, 3)$. True

In Exercises 45–50, use the graph to estimate the limits and valu the function, or explain why the limits do not exist.

45.



(a) $\lim_{x\to 3^-} f(x)$ 3

(b) $\lim_{x\to 3^+} f(x)$ −2

(c) $\lim_{x\to 3} f(x)$ No limit

(d) $f(3)$ 1

46.

(a) $\lim_{t\to -4^-} g(t)$ 5

\\STAFF04.SLCSD.NET\STAFF\rh031710\Documents\SamsungTechwin\SDPViewer\AVI

(33) $\lim_{x\to 0} \frac{\sin^2 x}{x}$ → $f(0) = \frac{\sin^2(0)}{0} = \frac{0}{0}$

graphically $\lim_{x\to 0} f(x) = 0$

$\lim_{x\to 0}\left[(\sin x)\left(\frac{\sin x}{x}\right)\right] = \left(\lim_{x\to 0} \sin x\right)\left(\lim_{x\to 0} \frac{\sin x}{x}\right) =$ (→ 0) (→ 1)

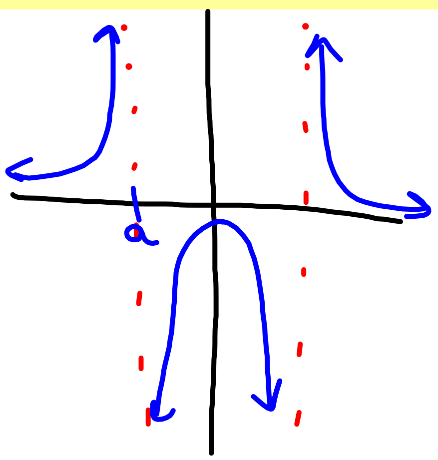
$0 \cdot 1 = \underline{\underline{0}}$

Vertical Asymptote

If the values of a function $f(x)$ outgrow all positive bounds as x approaches a finite number a, we say that $\lim_{x \to a} f(x) = \infty$. If the values of f become large and negative, exceeding all negative bounds as x approaches a finite number a, we say that $\lim_{x \to a} f(x) = -\infty$.

The line $x = a$ is a **vertical asymptote** of the graph of a function $y = f(x)$ if either

$$\lim_{x \to a^+} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a^-} f(x) = \pm\infty$$



You try...

Find the vertical asymptotes of the graph of $f(x)$ and describe the behavior of $f(x)$ to the right and left of each vertical asymptote.

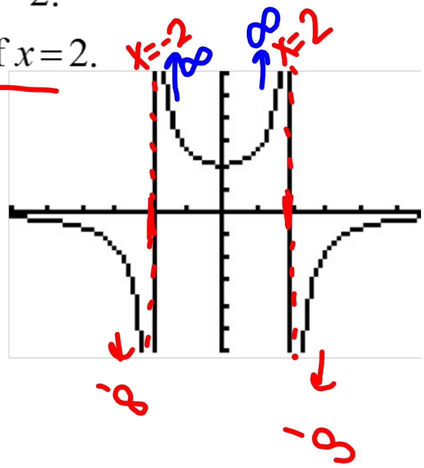
$$f(x)=\frac{8}{4-x^2} = \frac{8}{(2-x)(2+x)}$$

The values of the function approach $-\infty$ to the left of $x=-2$.

The values of the function approach $+\infty$ to the right of $x=-2$.

The values of the function approach $+\infty$ to the left of $x=2$.

The values of the function approach $-\infty$ to the right of $x=2$.



$$\lim_{x\to -2^-}\frac{8}{4-x^2}=-\infty \quad \text{and} \quad \lim_{x\to -2^+}\frac{8}{4-x^2}=+\infty$$

$$\lim_{x\to 2^-}\frac{8}{4-x^2}=+\infty \quad \text{and} \quad \lim_{x\to 2^+}\frac{8}{4-x^2}=-\infty$$

So, the vertical asymptotes are $x=-2$ and $x=2$

Sep 16-3:28 PM

Limits as $x \to \pm\infty$

The line $y = b$ is a **horizontal asymptote** of the graph of a function $y = f(x)$ if either

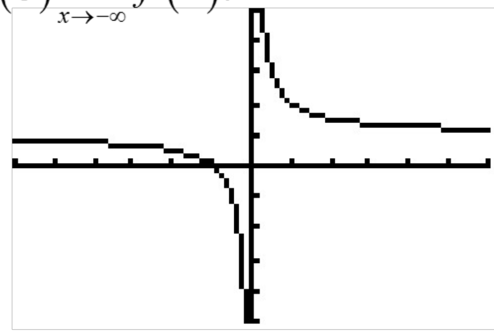
$$\lim_{x \to \infty} f(x) = b \quad \text{or} \quad \lim_{x \to -\infty} f(x) = b$$

You try...

Use a graph and tables to find (a) $\lim_{x \to \infty} f(x)$ and (b) $\lim_{x \to -\infty} f(x)$.

(c) Identify all horizontal asymptotes.

$$f(x) = \frac{x+1}{x}$$



a) $\lim_{x \to \infty} f(x) = 1$

b) $\lim_{x \to -\infty} f(x) = 1$

(c) Identify all horizontal asymptotes. $y = 1$

X	Y1
-150	.99333
-100	.99
-50	.98
0	ERROR
50	1.02
100	1.01
150	1.0067

Y1=(X+1)/X

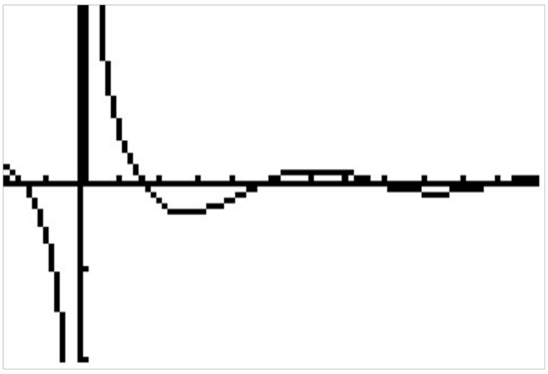
$f(x) \to 1$

$f(x) \to 1$

Sandwich Thm Revisited...

The sandwich theorem also holds for limits as $x \to \pm\infty$.

Find $\lim\limits_{x\to\infty} \dfrac{\cos x}{x}$ graphically and using a table of values.



X	Y1
-100	-.0086
100	.00862
300	-7E-5
500	-.0018
700	-.0012
900	7.4E-5
1100	8.2E-4

Y1=(cos(X))/X

The graph and table suggest that the function oscillates about the x-axi

Thus $y = 0$ is the horizontal asymptote and $\lim\limits_{x\to\infty} \dfrac{\cos x}{x} = 0$

Sep 16-3:32 PM

Properties of Limits as $x \to \pm\infty$

If L, M and k are real numbers and

$$\lim_{x\to\pm\infty} f(x) = L \quad \text{and} \quad \lim_{x\to\pm\infty} g(x) = M, \text{ then}$$

1. *Sum Rule*: $\lim_{x\to\pm\infty} \left(f(x)+g(x)\right) = L+M$

   The limit of the sum of two functions is the sum of their limits.

2. *Difference Rule*: $\lim_{x\to\pm\infty} \left(f(x)-g(x)\right) = L-M$

   The limit of the difference of two functions is the difference of their limits

3. *Product Rule*: $\lim_{x\to\pm\infty} \left(f(x)\bullet g(x)\right) = L\bullet M$

   The limit of the product of two functions is the product of their limits.

4. *Constant Multiple Rule*: $\lim_{x\to\pm\infty} \left(k\bullet f(x)\right) = k\bullet L$

   The limit of a constant times a function is the constant times the limit of the function.

5. *Quotient Rule*: $\lim_{x\to\pm\infty} \frac{f(x)}{g(x)} = \frac{L}{M},\ M \neq 0$

   The limit of the quotient of two functions is the quotient of their limits, provided the limit of the denominator is not zero.

6. *Power Rule*: If r and s are integers, $s \neq 0$, then

   $$\lim_{x\to\pm\infty} \left(f(x)\right)^{\frac{r}{s}} = L^{\frac{r}{s}}$$

   provided that $L^{\frac{r}{s}}$ is a real number.

   The limit of a rational power of a function is that power of the limit of the function, provided the latter is a real number.

Sep 16-3:33 PM

End Behavior Models

The function g is

(a) a **right end behavior model** for f if and only if $\lim_{x\to\infty}\frac{f(x)}{g(x)}=1.$

(b) a **left end behavior model** for f if and only if $\lim_{x\to-\infty}\frac{f(x)}{g(x)}=1.$

You try...

Find an end behavior model for

$$f(x)=\frac{3x^2-2x+5}{4x^2+7}$$

$g(x)=\frac{3x^2}{4x^2}$

Notice that $3x^2$ is an end behavior model for the numerator of f, and $4x^2$ is one for the denominator. This makes

$$\frac{3x^2}{4x^2}=\frac{3}{4}\text{ an end behavior model for } f.$$

In this example, the end behavior model for f, $y=\frac{3}{4}$ is also a horizontal asymptote of the graph of f. We can use the end behavior model of a rational function to identify any horizontal asymptote.
A rational function always has a simple power function as an end behavior model.

End Behavior Models

If one function provides both a left and right end behavior model, it is simply called an **end behavior model**.

In general, $g(x)=a_n x^n$ is an end behavior model for the polynomial function $f(x)=a_n x^n + a_{n-1}x^{n-1} + ... + a_0, \quad a_n \neq 0$

Overall, all polynomials behave like monomials.

Sep 16-3:35 PM

Homework

2.2 pgs.76 #3-45 (X3)

Oct 21-8:47 AM