Use a calculator to approximate each to the nearest thousandth.

1) $\log_3 4.8 = 1.428$

2) $\log_3 34 = 3.210$

3) $\log_5 6.68 = 1.180$

4) $\log_4 2.5 = 0.661$

5) $\log_6 2.1 = 0.414$

6) $\log_5 28 = 2.070$

7) $\log_5 27 = 2.048$

8) $\log_7 34 = 1.812$

9) $\log_6 1 = 0$

10) $\log_6 2.2 = 0.440$

change of base

$$\log_3 4.8 = \frac{\log 4.8}{\log 3} = \frac{\ln 4.8}{\ln 3}$$

2.1 Log Logic

A Develop Understanding Task

We began thinking about logarithms as inverse functions for exponentials in Tracking the Tortoise. Logarithmic functions are interesting and useful on their own. In the next few tasks, we will be working on understanding logarithmic expressions, logarithmic functions, and logarithmic operations on equations.

We showed the inverse relationship between exponential and logarithmic functions using a diagram like the one below:

Input → Output

$x = 3$ → $2^3 = 8$ → 3

$f(x) = 2^x$ $\qquad$ $\boxed{f^{-1}(x) = \log_2 x}$

We could summarize this relationship by saying:

$2^3 = 8$ so, $\log_2 8 = 3$

Logarithms can be defined for any base used for an exponential function. Base 10 is popular. Using base 10, you can write statements like these:

$10^1 = 10$ so, $\log_{10} 10 = 1$

$10^2 = 100$ so, $\underbrace{\log_{10} 100}$ $= 2$ what power of 10 is 100?

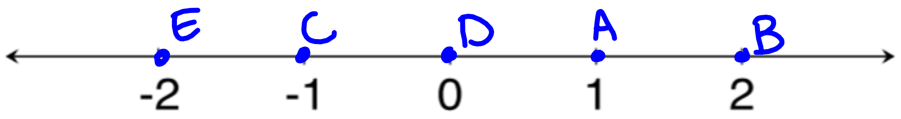
$10^3 = 1000$ so, $\log_{10} 1000 = 3$

The notation is a little strange, but you can see the inverse pattern of switching the inputs and outputs.

The next few problems will give you an opportunity to practice thinking about this pattern and possibly make a few conjectures about other patterns that you may notice with logarithms.

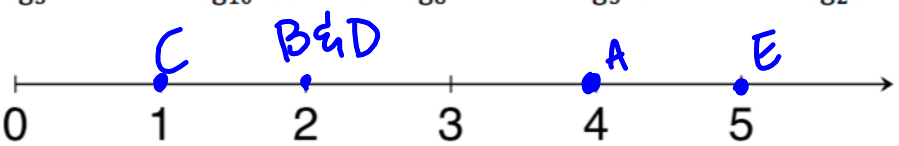
Place the following expressions on the number line. Use the space below the number line to explain how you knew where to place each expression.

1. A. $\log_3 3$ B. $\log_3 9$ C. $\log_3 \frac{1}{3}$ D. $\log_3 1$ E. $\log_3 \frac{1}{9} = -2$



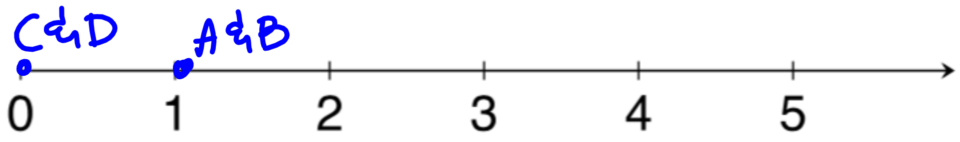
Explain: convert the log into its exponential form.

2. A. $\log_3 81$ B. $\log_{10} 100$ C. $\log_8 8$ D. $\log_5 25$ E. $\log_2 32$



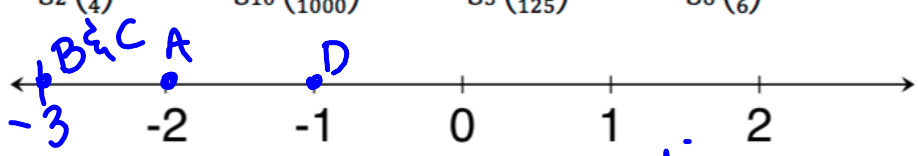
Explain: Ditto :)

3. A. $\log_7 7$ B. $\log_9 9$ C. $\log_{11} 1$ D. $\log_{10} 1$



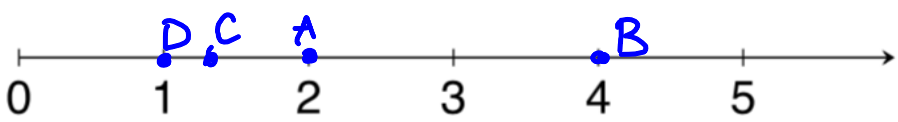
Explain: Ditto

4. A. $\log_2\left(\frac{1}{4}\right)$ B. $\log_{10}\left(\frac{1}{1000}\right)$ C. $\log_5\left(\frac{1}{125}\right)$ D. $\log_6\left(\frac{1}{6}\right)$



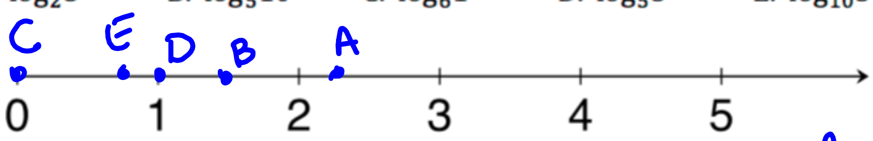
Explain: ditto

5. A. $\log_4 16$ B. $\log_2 16$ C. $\log_8 16$ D. $\log_{16} 16$



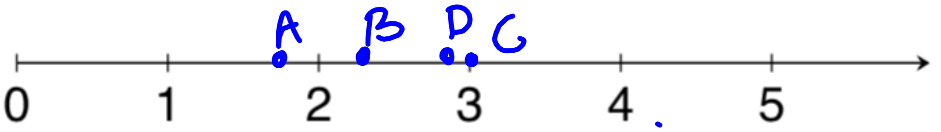
Explain: ditto & calculator

6. A. $\log_2 5$ B. $\log_5 10$ C. $\log_6 1$ D. $\log_5 5$ E. $\log_{10} 5$



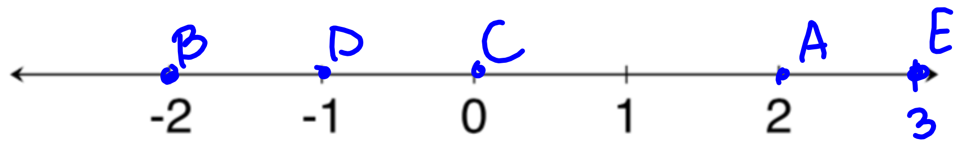
Explain: calculator

7. A. $\log_{10}50$ B. $\log_{10}150$ C. $\log_{10}1000$ D. $\log_{10}500$



Explain: ditto

8. A. $\log_3 3^2$ B. $\log_5 5^{-2}$ C. $\log_6 6^0$ D. $\log_4 4^{-1}$ E. $\log_2 2^3$



Explain: ditto

Based on your work with logarithmic expressions, determine whether each of these statements is always true, sometimes true, or never true. If the statement is sometimes true, describe the conditions that make it true. Explain your answers.

9. The value of $\log_b x$ is positive, $b>1$

Explain: Sometimes, $\log_b x$ will be negative when $0<x<1$ and positive when $x>1$.

10. $\log_b x$ is not a valid expression if x is a negative number.

Explain: Always true

~~$\log_2(-8)$~~ invalid

11. $\log_b 1 = 0$ for any base, b > 1.

always

Explain: Always $(b^0 = 1)$

12. $\log_b b = 1$ for any b > 1.

$\log_2 2$ $\log_5 5$
$\log_3 3$
$\log_4 4$

Explain: Always

$\log_2 1 = \log_3 1$
$\log_2 6 > \log_3 6$
$\log_2\left(\frac{1}{4}\right) > \log_3\left(\frac{1}{4}\right)$
-2 -1.26

13. $\log_2 x < \log_3 x$ for any value of *x*.

Explain:

14. $\log_b b^n = n$ for any b > 1.

Explain: Always

Homework

2.1 "Ready, Set, Go"