Section 1.3 & 1.5 Exponents & Logarithms

An exponential function looks like...

Parent Function	Graph	Domain	Range	Even/Odd	Transformations

Rules for exponents:

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EXAMPLES

1. Simplify $\dfrac{\left(-4x^2y^3z\right)^2}{6xz^{-2}}$

2. Rewrite 9^{2x} so it has a base of 3.

3. Solve $9^{2x}=3^{3x-5}$

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Applications:

Interest:

Exponential growth:

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EXAMPLE
The table shows the world population for several years.

	Year	Population (millions)
x=0	1986	4936
=1	1987	5023
=2	1988	5111
=3	1989	5201
=4	1990	5329
	1991	5422

Predict the world population in the year 2020.

$y = 4936(1.02)^x$

$x = 34$

$y = 4936(1.02)^{34}$

$y = 9678$ millons

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Exponential decay:

$$y = k \cdot a^x, \quad k > 0$$

$$0 < a < 1$$

EXAMPLE

The half-life of a radioactive substance is 20 days. There are 5 grams present initially. When will there be 1 gram present?

After 20 days

$$5\left(\tfrac{1}{2}\right)^1 = \tfrac{5}{2}\,g$$

After 40 days

$$5\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right) = \tfrac{5}{4}\,g$$

$$5\left(\tfrac{1}{2}\right)^2$$

$$y = 5\left(\tfrac{1}{2}\right)^{t/20}$$

$$1 = 5\left(\tfrac{1}{2}\right)^{t/20}$$

$$\tfrac{1}{5} = \tfrac{1}{2}^{t/20}$$

$$\log \tfrac{1}{5} = \tfrac{t}{20} \log \tfrac{1}{2}$$

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$$\log \tfrac{1}{5} = \tfrac{t}{20} \log \tfrac{1}{2}$$

$$20 \log \tfrac{1}{5} = t \log \tfrac{1}{2}$$

$$\frac{20 \log \tfrac{1}{5}}{\log \tfrac{1}{2}} = t$$

$$46.4 \approx t$$

days

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The number e: 2.71828 . . .

irrational

Euler's #

Logarithmic functions:

$\log_a x = y \qquad a^y = x$

Evaluate: $\log_2 32 = 5$ $\qquad \log_3 \frac{1}{27} = -3$ $\qquad \log_7 1 = 0$

$2^? = 32$

Commonly used logarithms:

$\log_e x = \ln x \qquad \log_{10} x = \log x$

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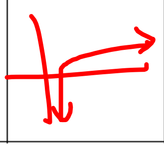
Parent Function	Graph	Domain	Range	Even/Odd	Transformations
$\log x$		$(0, \infty)$	$(-\infty, \infty)$	neither	rotate reflect translate dilate

Properties of logarithms: pg 40

base a: $a^{\log_a x} = x$ & $\log_a a^x = x$, $a > 1$, $x > 0$

base e: $e^{\ln x} = x$ & $\ln e^x = x$, $x > 0$

$x > 0, y > 0$

- product rule: $\log_a (xy) = \log_a x + \log_a y$
- quotient rule: $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
- power rule: $\log_a x^y = y \log_a x$

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EXAMPLES

Expand $\log\left(\frac{x^2}{(x-1)^3}\right)$

Condense (write as a single logarithm) $\frac{2}{3}\ln 8 - \ln\left(3^4 - 8\right)$

Solve $\log_4(x+3) + \log_4(2-x) = 1$

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Change of base formulas:

EXAMPLES

Graph $f(x) = \log_2 x$

Evaluate $\log_7 16$

Using logarithms to solve exponential equations:

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EXAMPLES

$e^{0.05t} = 3$

Sarah invests $1000 in an account that earns 5.25% compounded annually. How long will it take the account to reach $2500?

Using properties of logarithms to solve problems.

Solve for y: $\ln y = 2t + 4$

Solve for x: $\frac{5^x - 5^{-x}}{5^x + 5^{-x}} = \frac{1}{8}$

Solve for x: $\log_2(\log_2 x) = 2$

Solve for x: $(\log_3 x)^2 - \log_3 x^2 = 3$

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Homework

1.3 pg.26-27 #1-29EOO

1.5 pg.43 #1-15,29-45 odds

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